The Econometrics of Imperfect Knowledge Economics

– A Simulation Study of a Simple Model of Stock Prices and Earnings with Cointegrated VAR Estimations[†]

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This paper is an abridged version of Frydman, Goldberg, and Tabor (2013).

1 Introduction

A core premise of contemporary economic models is that researchers can adequately specify in probabilistic terms how individuals alter the way they make decisions and how the processes underpinning market outcomes unfold over time. Based on this core premise individual and aggregate outcomes at all points in time are represented with an overarching probability distribution. We refer to such models as determinate. To confront determinate models with empirical evidence a 'theory-first approach' to econometrics is typically used, see Hoover (2006*b*) and Spanos (2009). In the 'theory-first approach' the determinate theoretical model delivers a complete stochastic specification that relates aggregate outcomes to a set of explanatory variables, and the role of econometrics is solely to quantify the theoretical parameters of interest and test their statistical significance using regression or other statistical techniques (Spanos, 2006).

By contrast, the Imperfect Knowledge Economics (IKE) approach recognizes that an overarching stochastic specification of aggregate outcomes is beyond our reach because the process underpinning market outcomes is contingent: it changes at times and in ways that no one can fully anticipate with a probability distribution. Hence, theoretical IKE models are by design contingent and partly open: they allow for unanticipated change in the causal structure, which cannot be specified in advance in probabilistic terms. Confronting IKE models with empirical evidence is a challenge due to the contingency, as they do not imply an overarching causal structure for aggregate outcomes that can be directly estimated and tested using standard econometric tools.

In this project, we address this challenge. We show how cointegrated vector autoregressive (CVAR) models and extensions thereof, together with an underlying methodological framework that we call the Copenhagen-LSE-Oxford (CLO) approach, can serve as a basis for an IKE econometric methodology[1].

The CVAR model and CLO methodology is based on a 'data-first approach' to econometrics, where the stochastic specification of the econometric model is derived from the data based on statistical testing, rather than imposed from the outset based on *a priori* assumptions of a determinate theoretical model. Although we point out

[1] What we call the Copenhagen-LSE-Oxford (CLO) methodology originated from the work of Dennis Sargan at London School of Economics, but is today mainly associated with the work of David Hendry and co-workers at Oxford University and Søren Johansen and Katarina Juselius at the University of Copenhagen. For a broad introduction and discussion of the main econometric methodologies see Hoover (2006*a*), and see Hoover (2006*b*) and Spanos (2009) for a discussion of the 'data-first' and 'theory-first' approaches.

For a broad introduction to the theory and application of the CVAR model see Juselius and Johansen (2006), Johansen (1996), and Juselius (2006). Hendry (1995) provides a broad introduction to econometric modeling based on a general-to-specific approach, see also Mizon (1995) for a survey.

that *a priori* considerations based on economic theory is used as a guide in the variable selection, specification, and testing of the econometric model.

The essence of the 'data-first approach' to econometrics is the general-to-specific approach, which first seeks a general unrestricted model as a valid statistical representation of the data for the sample period considered, and then tests restrictions on the general model with the aim of finding a specific model that accounts for the information of the general model more parsimoniously. The first step involves what Spanos (2010) calls 'statistical testing,' which focuses on testing the statistical adequacy of the model as a representation of the data for the sample period considered. Once a valid statistical representation of the data is found, the empirical validity of potentially conflicting hypotheses from economic theory can be imposed and tested as restrictions and reductions of the general model, which involves what Spanos (2010) calls 'substantive testing.'

The 'data-first approach' is suitable for empirically testing IKE models as it allows for contingency in the underlying data-generating process by searching for stochastic specifications as statistically valid 'local' representations of the data. Because structural change cannot be modeled as probabilistic, the specification of an IKE econometric model has to be based upon and tested against the data. Consequently, the 'data-first' methodology of the CVAR model allows for structural change to be identified *ex post* in the historical data without an *ex ante* probabilistic specification of exactly when and how the structural breaks occur. The key point here is that while IKE acknowledges that an economist cannot fully specify the occurence of structural breaks in an economic model *ex ante*, an econometrician using the 'data-first approach' can test for and identify structural breaks in the historical economic data *ex post*. However, it should be noted that IKE does not imply that there are no empirical relations that are stable at the aggregate level over time; IKE just don't start from an *a priori* assumption that all empirical relations are indeed stable at all points in time.

The CVAR model's system approach and its distilling of time series according to their degree of persistence has proven to be extremely useful for representing and modeling non-stationary macroeconomic and financial data. In practice, the specification of a statistically well-specified CVAR model requires selecting a suitable lag-length, including level shifts and dummy variables, and potentially splitting the sample into subsamples with different CVAR models for each subsample. With good econometric modeling skills, and a sense of the context under study, an econometrician can identify samples of historical data in which a specific CVAR model adequately represents the data. Inference in the CVAR model is then valid and testable hypotheses based on an IKE model can be tested as restrictions on the general model.

Although the 'data-first approach' of the CVAR provides a suitable way to empirically estimate and test IKE models, there are important challenges in bridging the empirics and theory. First, IKE models allow for contingent structural change in the stochastic representations of the individual variables and the causal parameters, whereas a standard CVAR model has parameters which are assumed constant over time. An important question here is under what conditions can structural breaks in both the stochastic representations of the variables and the causal parameters be identified using standard statistical procedures and residual misspecification tests. Moreover, to what extent and under what conditions can we determine whether seperate subsample analyses are preferred over a full-sample analysis. Finally, many IKE models imply that markets are boundedly unstable: wide price swings away from benchmark values are eventually reversed and sustained movements back towards these values occur.[2] This implication suggests that there may be a connection between the boundedness of the market process and our ability to estimate cointegration relationships using the CVAR model. For example, is it the case that a greater tendency for reversals in the market leads to a greater chance that the system will be characterized by cointegration relationships?

To analyze these and other questions, this project simulates outcomes from IKE models and uses CVAR models to analyze the simulated data econometrically. In this paper, we present some preliminary results based on simulations from a simple IKE model of stock prices. There are two key features of this model that underpin our results: i) there are stretches of time in which market participants either maintain their forecasting strategies or revise them only moderately, and ii) price swings away from the benchmark value are bounded. In modeling this second feature, both the stock price and current earnings are assumed to fluctuate persistently, but boundedly so, around a common long-run trend in earnings.

Our simulations show that even though the specification of the CVAR model is 'wrong' compared to the specification used to simulate outcomes from an IKE model, it can nonetheless be used as a statistically adequate representation of the data with an adequate lag structure. Furthermore, we show that the bounded instability of the relationship between the simulated asset prices and earnings plays a key role in our ability to understand and interpret the estimates of the CVAR model. Cointegration between the simulated time-series can be found when the time-varying parameters of the simulated series are bounded, which implies that the variables are 'stochastically

[2]For example, in the Frydman and Goldberg (2007, 2013) model of asset price swings and risk, persistent trends in fundamental variables and the influence of psychological and social factors can lead market participants to bid asset prices persistently away from benchmark values over a stretch of time. But, this instability is bounded in part because of risk considerations: if departures from bechmark values continued to grow, they would eventully lead market participants to revise their forecasting strategies in ways that resulted in a sustained countermovement back toward benchmark values.

cointegrated', rather than cointegrated. In this case, the CVAR model can be used to estimate the unconditional sample means of the time-varying parameters, see Tabor (2013).

Future research will extend the simulations to handle more general IKE specifications. Moroever, the econometric analyses of this research will focus directly on identifying and testing for structural breaks in the CVAR model and on deriving distributions of the estimators from the simulations, which can be compared to the standard distributions of the CVAR model.

The rest of the paper is structured in the following way. In section 2 a simple IKE model of stock prices and earnings is presented and the link between boundedness in the model and cointegration is discussed. Section 3 presents the simulation setup for the simple IKE model, introduces the CVAR model used for the econometric analysis of the simulated data, shows an illustration of the simulated series, and finally presents and discuss the results from the CVAR estimations based on the simulated data.

2 A Simple IKE Model of Stock Prices and Earnings

In this paper we consider a simple version of an IKE model of stock prices and earnings. The model a simplified version of the general IKE model of asset price swings and risk presented in Frydman and Goldberg (2007, 2013). The simple model considered here captures some—but not all—of the main ideas of an IKE asset pricing model in a simple way that mimics some of the key features of the stock market. The simple model considered here allows us to simulate potential outcomes from an IKE model which can be econometrically analysed with a CVAR model in a fairly simple setup. It serves as a starting point for the research project and it will be gradually extended. For a full presentation and discussion of the general IKE model of asset price swings and risk, see Frydman and Goldberg (2013).

The general IKE model of long swings in asset prices can be written in reduced form at the aggregate level as:

$$p_t = \widehat{p}_{t|t+1} - \widehat{up}_t + \varepsilon_{p,t}, \tag{1}$$

where p_t is the asset price at time t, $\widehat{up}_t$ is an uncertainty premium, and $\varepsilon_{p,t}$ is a normal IID error term with variance σ_p^2.

A key element of the IKE asset pricing model is the assumption that the uncertainty premium covaries positively over time with the gap between the asset price and a historical benchmark level. Defining this gap as:

$$gap_t = p_t - p_t^{BM}, \tag{2}$$

the uncertainty premium can be represented as:

$$\widehat{up}_t = \sigma \cdot gap_t = \sigma \left(p_t - p_t^{BM}\right), \tag{3}$$

where p_t^{BM} is the benchmark level for the asset price, and $\widehat{p}_{t|t+1}$ is a representation of the aggregate forecast of the future asset price. The parameter σ determines the effect of the gap on the asset price, and here we assume for simplicity that the parameter is constant.

Given the specification of the uncertainty premium the asset price can be written as:

$$p_t = \widehat{p}_{t|t+1} - \sigma \left(p_t - p_t^{BM}\right) + \varepsilon_t^p, \tag{4}$$

which is equivalent to

$$p_t = \lambda \widehat{p}_{t|t+1} + (1 - \lambda) p_t^{BM}, \tag{5}$$

and

$$p_t = p_t^{BM} + \lambda \left(\widehat{p}_{t|t+1} - p_t^{BM}\right). \tag{6}$$

Equation (5) shows that in each period the asset price is represented as a weighted average of the price forecast and the benchmark price with weights given by $\lambda = 1/ (1 + \sigma)$ and $1 - \lambda$, respectively. Equation (6) shows that the asset price can also be represented as the benchmark price plus a multiple of the deviation between the forecasted price and the benchmark price.

The overall idea of the IKE model of asset price swings and risk is that market participants base their forecasting strategies of the future price on a combination of fundamental, psychological, and social factors, and that persistent trends in the fundamental variables and the influence of psychological and social factors can lead market participants to bid asset prices persistently away from the benchmark price over a strech of time. However, this instability is bounded in part because of risk considerations: if departures from benchmark values continued to grow, they would eventually lead market participants to revise their forecasting strategies in ways that resulted in a sustained countermovement back towards benchmark values.

The bounded instability implies that the price forecast $\widehat{p}_{t|t+1}$ is allowed to move persistently away from the benchmark price level p_t^{BM}, but ultimately such movements are bounded. Hence, from equation (6) it follows that the asset price p_t moves persistently, but boundedly so, around the benchmark price p_t^{BM}.

2.1 A Representation of the Benchmark Price and Price Forecast

We now depart from the general IKE model of asset price swings and risk and consider a simple model of stock prices and earnings. In this simple model both the benchmark price and the representation of the aggregate price forecast depend only on corporate earnings, and thereby the stock price is assumed only to depend on corporate earnings. We assume that earnings has a long-run non-stationary trend and a short-run component fluctuating persistently around the long-run trend. The benchmark price depends on the long-run trend in earnings, while the price forecast for simplicity is represented only in terms of currently observed earnings.

First, we assume that there is a non-stationary long-run trend in earnings which can be represented as a random walk:

$$\overline{x}_t = \overline{x}_{t-1} + \mu_x + \varepsilon_{\overline{x},t}, \tag{7}$$

where $\mu_x > 0$ is a constant positive drift term and $\varepsilon_{\overline{x},t}$ is an IID normal error with variance $\sigma^2_{\overline{x}}$. We assume that current earnings x_t fluctuate persistently around the long-run trend $\overline{x}_t$, and that the fluctuations can be represented by a segmented trend specification. The segmented trend push current earnings persistently away from the long-run trend, but eventually a reversal in the segmented trend occurs, thereby causing a countermovement of current earnings back towards the long-run trend. As current earnings reach the long-run trend level they are allowed to continue away from the long-run trend in the opposite direction, but eventually another reversal will cause another countermovement back towards the long-run trend. Hence, the idea is that the short-run fluctuations in earnings are bounded around the long-run trend, so that current earnings has a non-stationary long-run trend and a bounded short-run trend represented with a segmented trend specification.

To capture this idea, we assume that current earnings can be represented as stationary around a segmented trend:

$$x_t = (1 - \rho_x)\,\Psi_t + \rho_x x_{t-1} + \varepsilon_{x,t}, \tag{8}$$

where Ψ_t is a segmented trend, $0 < \rho_x < 1$ is an autoregressive constant parameter, and $\varepsilon_{x,t}$ is a normal IID error term with variance σ^2_x. Moreover, we assume that the variance of current earnings is greater than the variance of the long-run trend in earnings, $\sigma^2_x > \sigma^2_{\overline{x}}$.

The segmented trend Ψ_t has a number, n, of long swings and we let $0 = T^*_0 < T^*_1 < T^*_2 < ... < T^*_n = T$ denote the points in time at which the segmented trend changes direction. The length of the ith swing is given by $T_i = T_i^* - T_{i-1}^*$. The segmented trend can be written as:

$$\Delta\Psi = \mu_t \qquad \text{for } t = T_{i-1}^*, ..., T_i^*, \tag{9}$$

where μ_t is restricted to take on values with opposite signs in subsequent segments. The IKE model does not specify when the switches in μ_t occur and what values it can take on with a probability distribution. Though, we assume that the probability of a switch in the direction of the segmented trend increases with the deviation between current earnings and the long-run trend, $x_t - \overline{x}_t$, so that the deviation is bounded through the segmented trend specification.

We can rewrite current earnings as:

$$x_t = \overline{x}_t + (\Psi_t - \overline{x}_t) + \rho_x (x_{t-1} - \Psi_t) + \varepsilon_{x,t}, \tag{10}$$

which shows that current earnings has a non-stationary long-run component determined by $\overline{x}_t$, a bounded component determined by the deviation between the segmented trend and the long-run trend $(\Psi_t - \overline{x}_t)$, and finally a stationary component. The important implication of the assumed specification is that the deviation between current earnings and their long-run trend is bounded.

Next, the uncertainty premium depends on the gap between current stock price and the benchmark price, which we assume can be represented as a multiple of the long-run trend in earnings:

$$p_t^{BM} = B'\overline{x}_t, \tag{11}$$

where B is a parameter assumed to be constant for the sample considered, which however need not need be the case in a general IKE model.

Finally, we represent the aggregate forecast of the future price as:

$$\widehat{p}_{t|t+1} = b_t' x_t, \tag{12}$$

where we assume for simplicity that the aggregate forecasts of the future price can be represented only in terms of current earnings x_t, so that b_t is a scalar representing the weight attached to earnings in the forecasting strategy.

Movements in the price forecast depend on two factors, movements in earnings and revisions of the forecasting strategies:

$$\Delta\widehat{p}_{t|t+1} = b_{t-1}' \Delta x_t + \Delta b_t' x_t. \tag{13}$$

In modeling revisions of the forecasting strategies we impose the 'guardedly moderate revisions' conditions of Frydman and Goldberg (2007, 2013), which restrict the changes in the forecasting weights b_t so that the impact of revisions on the total change in the price forecast is smaller than the impact of the segmented trend in earnings[3]:

$$|\Delta b_t' x_t| < \delta_t, \tag{14}$$

where $|\cdot|$ denotes an absolute value and

$$\delta_t = \left| b_{t-1}' \mu_t \right| \tag{15}$$

represents the 'baseline trend' in the price forecast which would occur on average over the period from T_i^* to T_{i-1}^* if the forecasting strategies were not revised, i.e. if $\Delta b_t = 0$, over the period. The condition embodies the idea that if individuals revise their forecasting strategies, they are reluctant to do so in ways that would change their price forecasts too much from what would be associated with no revision at all.

As the long-run and segmented trends in earnings unfold over time, the way they feed into the price forecast and the stock price depends on the revisions of the forecasting strategies. During streches of time where the guardedly moderate revisions hold, the revisions can either reinforce or impede the trends in earnings. Thus, we can think of the revisions of the forecasting weight to current earnings as representing how market participants interpolate the trends in earnings into the future; if, for example, market participants forecast that an upward current trend in earnings is unsustainable, so that they expect a reversal some time in the near future, they might revise their forecasting strategies in impeding ways, so that the impact of their revisions counteract the current trend in earnings. Likewise, market participants forecasting that a downward current trend will soon be reversed might revise their forecasting strategies in reinforcing ways.

Based on this interpretation of the simple IKE model of stock prices and earnings, it can be interpreted as equivalent to the present-value model of Barsky and De Long (1993), with earnings taking the role of dividends in their model. In the Barsky and DeLong present value model a small part of the shocks to dividends in each period feeds into the future growth rate of dividends, which changes the present value of the future dividends that determines the stock price. However, while the shocks to dividends feed into both current dividends and the growth rate the determinate model assumes that the impact on the stock price is constant over time. By contrast, this simple IKE model acknowledge that the way the trends in earnings feed into the stock price depends on how market participants revise their forecasting strategies.

Moreover, the contingency of an IKE model allows for non-moderate revisions of the forecasting strategies at points in time that cannot be specified with a probability

[3] We impose only the first of the two condition specified in Frydman and Goldberg (2007, 2013) as the second becomes redundant in the univariate case we are considering here.

distribution. Thus, at points in time that cannot be anticipated the revisions of the forecasting strategies are allowed not to fall within the the qualitative range specified by the condition in equation (14). However, we do impose the additional condition that whenever a non-moderate revision of the forecasting strategies occurs, the new set of forecasting weights b_t^{NM} fall within a qualitatively range which is symmetrically bounded around the parameter B:

$$\underline{b} < b_t^{NM} < \overline{b}, \tag{16}$$

where $\underline{b}= B - \tau_b$ and $\overline{b} = B + \tau_b$ represent the upper and lower bounds.

2.2 Bounded Instability and Cointegration Between Stock Prices and Earnings

Based on equation (6) and the above representations of earnings, the benchmark price, and the price forecast, the stock price can be written as:

$$\begin{aligned} p_t &= p_t^{BM} + \lambda\left(\widehat{p}_{t|t+1} - p_t^{BM}\right) + \varepsilon_{p,t} \\ &= B'\overline{x}_t + \lambda\left(b_t' x_t - B'\overline{x}_t\right) + \varepsilon_{p,t}. \end{aligned} \tag{17}$$

For the stock price to fluctuate boundedly around the benchmark level consistent with the long-run trend in earnings, the deviation between the price forecast and the benchmark price must be bounded. The simple representation considered here allows us to decompose this deviation into two components:

$$\begin{aligned} \widehat{p}_{t|t+1} - p_t^{BM} &= b_t' z_t - B' x_t \\ &= B'\left(x_t - \overline{x}_t\right) + \left(b_t - B\right)'\overline{x}_t, \end{aligned} \tag{18}$$

so that the boundedness of each of the two components can be considered individually.

First, the representation of earnings implies that the deviation between current earnings and the long-run trend in earnings $x_t - \overline{x}_t$ is bounded. The segmented trend cause current earnings to fluctuate persistently around the long-run trend $\overline{x}_t$, so even though the long-run trend $\overline{x}_t$ is non-stationary—and hence not bounded—the deviation between the two is bounded.

The second term in equation (18) is a product of $b_t - B$ and the non-stationary long-run trend in earnings $\overline{x}_t$. However, despite that $\overline{x}_t$ itself is not bounded, boundedness of $(b_t - B)'\,\overline{x}_t$ over time requires only that $b_t - B$ is bounded with mean zero over time. In that case the product of a mean zero bounded process and a non-bounded process will become bounded[4]. The guardedly moderate revisions condition in equation (14) does not imply boundedness of b_t, so to achieve boundedness of $b_t - B$ we assume that the probability of a non-moderate revision increases with the deviation $b_t - B$, and by restricting the non-moderate revision to the symmetric range around B, c.f. equation (16).

Based on the above specifications, each of the two terms $x_t - \overline{x}_t$ and $b_t - B$ are bounded:

$$x_t - \overline{x}_t \sim \text{bounded,} \tag{19}$$

$$b_t - B \sim \text{bounded,} \tag{20}$$

which implies that the deviation between the price forecast and the benchmark price is bounded:

$$\widehat{p}_{t|t+1} - p_t^{BM} \sim \text{bounded.} \tag{21}$$

It follows from equation (17) that fluctuations of the stock price are bounded around the benchmark price over time. Hence, the stock price can move persistently away from the benchmark price consistent with the long-run trend in earnings, as the segmented trend pushes current earnings away from their long-run trend, or as the forecasting strategies are revised in reinforcing ways so that b_t move away from B. Moreover, the two effects might impact the stock price in the same direction during some streches of time, while they might outweigh each other during other streches of time. However, movements in the stock price away from the benchmark price consistent with the long-run trend in earnings are ultimately bounded as a reversal in the segmented trend push current earnings back towards the long-run trend, or as market participants revise their forecasting strategies in non-moderate ways causing a reversal of the price forecast—and hence the stock price—back towards the benchmark price.

The important implication of the two boundedness conditions in equations (19) and (20) is that, despite the underlying bounded instability, the stock price and current earnings share a common trend given by the non-stationary long-run trend in earnings.

Because the stock price and earnings share a common trend we can think of them both as being cointegrated with the long-run trend in earnings—as well as with each other—though the specification of boundedness does not fully correspond to a standard cointegration relation. First, the boundedness between current earnings and their long-run trend can be thought of as a cointegration relation with a time-varying adjustment

[4]In a standard stochastic specification the multiplicative process of a stationary mean zero process and a non-stationary process becomes 'stochastically trendless', which means that the stochastically trendless property of the non-stationary process dominates the multiplicative process, see McCabe et al. (2003).

coefficient: when the segmented trend push earnings away from the long-run trend it corresponds to an equilibrium-increasing adjustment coefficient; and when the segmented trend push earnings towards the long-run trend it corresponds to an adjustment coefficient which is equilibrium-adjusting. Second, as the stock price fluctuates boundedly around the long-run trend in earnings we can think of the two as being cointegrated, with persistent deviations from the cointegration relation driven by the two bounded terms in equation (18).

As the stock price and current earnings fluctuate boundedly around the same common stochastic trend given by the non-stationary long-run trend in earnings, the deviation between them is bounded and we can think of the stock price and current earnings as being cointegrated. Though, the boundedness of the IKE model is based on a time-varying specification which differs from a standard stochastic specification of cointegrated relations. Moreover, the deviations from the common trend might be so persistent that the variables should be thought of as being cointegrated from $I(2)$ to $I(1)$, rather than from $I(1)$ to $I(0)$.

To see that the deviation between the stock price and current earnings is indeed bounded, rewrite equation (17) to:

$$p_t = b_t' x_t - (1-\lambda)\left(b_t' x_t - B'\overline{x}_t\right) + \varepsilon_{p,t}, \tag{22}$$

and re-arrange terms to get:

$$p_t - b_t' x_t = -(1-\lambda)\left(b_t' x_t - B'\overline{x}_t\right) + \varepsilon_{p,t}. \tag{23}$$

We know from above that the term $b_t' x_t - B'\overline{x}_t$ is bounded given the assumptions, so the right-hand side of equation (23) is bounded. Hence, the deviation between the stock price and the price forecast on the left-hand side is bounded. Furthermore, the assumption that $(b_t - B)$ is bounded with mean zero implies that the stock price p_t and current earnings x_t are bounded, and we can think of the stock price and current earnings as being 'stochastically cointegrated'[5]. By rewriting equation (23) as:

$$p_t - B' x_t = (b_t - B)' x_t - (1-\lambda)\left(b_t' x_t - B'\overline{x}_t\right) + \varepsilon_{p,t}. \tag{24}$$

[5]In a standard stochastic specification, stochastic cointegration between two variables requires that the time-varying cointegration parameter is stationary. Under this assumption the cointegration relation with time-varying cointegration parameter β_t can be written as:

$$\beta_t' X_t = \beta' X_t + (\beta_t - \beta)' X_t,$$

where the second term becomes 'stochastically trendless' when $\beta_t - \beta$ is stationary with unconditional mean 0, see McCabe et al. (2003) and Tabor (2013).

it can be seen that all terms on the right-hand side are bounded, once again because the term $(b_t - B)' x_t$ can be thought of as being 'stochastically trendless'. Thus, $p_t - B'x_t$ becomes bounded.

3 Simulations of the IKE Model of Stock Prices and Earnings

We use simulations to examine the link from the bounded instability of an IKE model to estimation of cointegration relations in a CVAR model. Thus, we simulate outcomes from the simple IKE model of stock prices and earnings and analyze the simulated data for the stock price and current earnings econometrically with the CVAR model.

The CVAR model is 'wrong' compared to the specification of the IKE model used to simulate the data: the specification of boundedness in the IKE model does not correspond to the specification of cointegration relations in a CVAR model, and the parameters of the IKE model are time-varying, while the CVAR model has constant parameters. The results in Tabor (2013) suggest that the CVAR model is a quite robust model to use even though there is an underlying bounded parameter instability in the data. Though, it is unclear if and under what conditions the CVAR model can be used to estimate empirical relations between variables based on the specification of an IKE model.

We address this question by using simulations, and here we present some preliminary results. We show that as long as the deviations of $x_t - \overline{x}_t$ and $b_t - B$ are not 'too large' the CVAR model can be used as a statistically valid representation of the data along the key dimensions, and we do find the simulated stock price and current earnings to be cointegrated—and hence sharing a common stochastic trend—with the estimated cointegration coefficients close to the coefficient B as we would expect based on equation (24).

3.1 The Simulation Design

An IKE model acknowledges contingent changes that cannot be specified in advance with a probability distribution. By contrast, computer simulations requires a deterministic or probabilistic specification of both when and how the contingent structural breaks occur. Though, while an IKE model itself cannot be simulated because it is contingent by design, we can simulate outcomes that are consistent with an IKE model, and using simulations we can easily check the robustness of a specific specification.

In the simulations presented here we use a standard logistic function to simulate the probabilities of a switch in the direction of the segmented trend and a non-moderate revision of the forecasting strategies, respectively, at each point in time. The logistic function has the form:

$$P\left(\text{break}_{i,t}\right) = \left[1 + \exp\left(-g_i\left(z_{i,t-1} - c_i\right)\right)\right]^{-1}, \tag{25}$$

where c_i is a threshold value where the probability of a break is one, g_i determines the curvature, and $z_{i,t-1} - c_t$ determines the distance that the probabilities depend on for $i = \mu, b$, which represent the probabilities of a break in the segmented trend and the forecasting weights, respectively.

We let the probability of a break in the segmented trend depend on the absolute deviation $x_{t-1} - \overline{x}_{t-1}$, so we set $z_{\mu,t} = |x_{t-1} - \overline{x}_{t-1}|$, and we let the probability of a non-moderate revision of the forecasting strategies depend on the absolute deviation $b_{t-1} - B$, so we set $z_{b,t} = |b_{t-1} - B|$. Thus, as $|x_{t-1} - \overline{x}_{t-1}|$ increases, the probability of a switch in the direction of the segmented trend increases, and eventually as $|x_{t-1} - \overline{x}_{t-1}| \geq c_\mu$ the probability of a switch reaches one. Moreover, after a switch in the direction of the segmented trend cause a countermovement in x_t towards $\overline{x}_t$, we set the probability of a switch in the direction to zero until x_t has crossed $\overline{x}_t$. Likewise, as $|b_{t-1} - B|$ increases, the probability of a non-moderate revision increases, and eventually the probability of a non-moderate revision reaches one as $|b_{t-1} - B| \geq c_b$.

At each point in time, we make two random draws from a standard uniform distribution at each point in time, and if they exceed the simulated probabilities we draw a new μ_t or b_t, respectively. The new μ_t is uniformly drawn within a specified range from $\underline{\mu}$ to $\overline{\mu}$, and with opposite sign compared to μ_{t-1}, while the new b_t^{NM} is drawn uniformly within the range from $B - \tau_b$ to $B + \tau_b$.

We fix the curvature parameters $g_\mu = g_b = 1.0$ and the threshold value for non-moderate revisions $c_b = 4.0$, and simulate the IKE model for different values of the threshold parameter g_μ and the range for non-moderate revisions τ_b. These two parameters are crucial determinants of the degree of bounded instability in the simulated system, and hence they are the parameters of greatest interest. The greater the threshold parameter c_μ, the greater deviation between current earnings and the long-run trend is allowed before a reversal eventually occurs. The parameter τ_b determines the symmetrical range around B within which a non-moderate revision is randomly drawn. The guardedly moderate revisions are symmetrical and by themselves do not ensure boundedness of b_t around B, so this boundedness occurs solely through the non-moderate revisions in this specification: if b_{t-1} is far above B, but below $B + \tau_b$, the probability of drawing a new b_t^{NM} below b_{t-1} is large.

The rest of the simulation setup follows the IKE model of stock prices and earnings as described above. We fix the following parameter values for all simulations presented here:

$$B = 2.0, \quad (26)$$

$$b_0 = 2.0, \quad (27)$$

$$\lambda = 0.5, \quad (28)$$

$$[\sigma_p, \sigma_x, \sigma_{\overline{x}}] = [0.5, 0.5, 0.1], \quad (29)$$

$$[p_0, x_0, \overline{x}_0] = 5.0, \quad (30)$$

$$\rho_x = 0.5, \quad (31)$$

$$\mu_{\overline{x}} = 0.01, \quad (32)$$

$$[\underline{\mu}, \overline{\mu}] = [0.02, 0.15] \quad (33)$$

$$[g_\mu, g_b] = 1.0, \quad (34)$$

$$c_b = 4.0. \quad (35)$$

We simulate time-series for p_t, x_t, and $\overline{x}_t$ for $i = 1, 2, ..., N$ different data-generating processes based on the $N = 16$ combinations of the parameters:

$$\tau_b^i \in \{0.25; 0.50; 1.00; 1.50\}, \quad (36)$$

and

$$c_z^i \in \{2.0; 3.0; 4.0; 5.0\},$$

where the upper limits for the two parameters are selected as the upper limits where cointegration appears to be found among the variables. For each of the i parameter specifications, $S = 1.000$ replications of time-series are simulated with the different sample lenghts t as given by:

$$t \in \{200; 400; 1.000\}, \quad (37)$$

so that in total 48.000 time-series are simulated. The simulations are performed in Ox, with a random seed set to 1.000 and reset for each new i, so that the random draws are the same across the different specifications.

For each simulation CVAR model is estimated for the simulated time-series for the stock price p_t and earnings x_t, and averages of the results over the $S = 1.000$ replications are reported for each data-generating process i and for the different sample lenghts.

3.2 An Illustration of the Simulated Series

The simulated outcomes for the specification $i = 7$, where $\tau_b = 0.5$ and $c_\mu = 4.0$, are shown in Figures 1.1 to 1.5 in the appendix.

Figure 1 shows the simulated series x_t and $\overline{x}_t$ in the upper panel, the gap between the two in the middle panel, and finally the simulated probabilities of a reversal in the segmented trend in the lower panel. From the upper panel the segmented trend specification of x_t around $\overline{x}_t$ is evident. Moreover, it should be noted that after each reversal in the segmented trend, a new value for μ_t is randomly drawn within a range, so the segmented trends have different slopes for the different segments. From the middle panel it can be seen how the gap between current earnings and their long-run trend is bounded over time, and in the lower panel it can be seen that the probability of a reversal increase as the segmented trend drives current earnings away from the long-run trend. The blue squares in the lower panel indicate the 23 reversals in the segmented trend over the sample.

Figure 2 shows the simulated weights attached to current earnings in the forecasting strategies. The upper panel shows the simulated weights b_t, along with B and indicators for non-moderate revisions. It is clear from the graphs that the simulated parameter b_t is boundedly unstable over time, and that the non-moderate revisions imply a number of large jumps in the forecasting weights. The middle panel shows the qualitative ranges for the guardedly moderate revisions, along with the simulated revisions of the forecasting strategies within these ranges. The lower panel shows the probabilities of a non-moderate revision of the forecasting strategies over time. On average this probability is around 2.5 percent, and over the long sample of 1.000 observations 22 revisions are simulated as non-moderate, as indicated by the blue squares.

Figure 3 shows the simulated stock price, the benchmark price, and the price forecast in the upper panel. As the stock price is represented as a weighted average of the benchmark price and the price forecasts it lies between the two over the entire sample period. The middle panel shows the deviation between the price forecast and the benchmark price, which is equivalent to a scaled version of the gap between the stock price and the benchmark price. The lower panel shows the decomposition of the deviation between the price forecast and the benchmark price as specified in equation (24). From the graphs it can be seen that each of the two components are bounded over the sample.

In Figure 4 the simulated stock price and current earnings are shown in the upper panel along with the simulated long-run trend in earnings. The middle panel shows the deviation $p_t - B'x_t$, while the lower panel show the deviation $p_t - B'\overline{x}_t$.

Finally, Figure 5 shows the first-differences of the stock price and current earnings in the two upper panels, and it can be seen that there are a few very large outliers, which are caused by the jumps in b_t due to non-moderate revisions. The lower graph displays the estimated cointegration relation, which looks almost identical with the graph in the lower panel in Figure 4. Despite the bounded instability in the relation between the stock price and earnings, a CVAR model for the series displayed in Figures 1-4 finds the two variables to be cointegrated with the estimated cointegration relation given by $\hat{\beta}' X_t = p_t - 2.38 \cdot x_t$.

4 The Cointegrated VAR Model

The p-dimensional vector autoregressive (VAR) model with k lags in error-correction form is given by:

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t, \tag{38}$$

for $t = 1, 2, ..., T$ and where $X_{-k-1}, ..., X_0$ are fixed. The error terms ε_t are assumed to be independent and Gaussian with mean zero and covariance Σ. The parameters Π and Γ_i are of dimension $(p \times p)$, the parameters μ_0 and μ_1 of dimension $(p \times 1)$. Dummy variables and mean shifts can be included in D_t, which has dimension $\left(p^D, 1\right)$, and the parameters Φ has dimensions $\left(p \times p^D\right)$.

The system is cointegrated if the matrix Π has reduced rank $r < p$, so that Π can be written as

$$\Pi = \alpha \beta', \tag{39}$$

where α and β are $(p \times r)$ matrices of full column rank, see Johansen (1996). Under the reduced rank condition the cointegrated VAR model is given by:

$$\Delta X_t = \alpha \beta' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t. \tag{40}$$

The levels X_t are nonstationary while the r cointegration relations $\beta' X_t$ are stationary. Hence, while the levels X_t are integrated of order one, $X_t \sim I(1)$, the linear combinations $\beta' X_t$ are integrated of order zero, $\beta' X_t \sim I(0)$, so that the process ΔX_t is stationary, $\Delta X_t \sim I(0)$. The cointegration relations determine the deviations from the long-run relations between the variables, while the α-coefficients measures the rate of adjustments to deviations from the long-run cointegration relations. For a full introduction to the theory and application of the CVAR model, see Johansen (1996) and Juselius (2006).

The notion of cointegration can be interpreted in the following way: there is a stationary long-run equilibrium relation between the non-stationary variables, and whenever the variables move away from this long-run relation at least one of the system variables is adjusting, so that the deviations from the long-run relations are stationary. Hence, even though the non-stationarity makes the individual system variables path-dependent, the cointegration relations ensure that the deviations between them are bounded.

Despite that the CVAR model has constant parameters, it has shown able to estimate the unconditional mean of time-varying cointegration parameters, at least in small systems. The requirement for this to be the case is that the time-varying parameters are stationary, so that the variables are 'stochastically cointegrated', rather than cointegrated with constant parameters. Disregarding deterministic terms and lagged first-differences, a stochastically cointegrated system can be written as:

$$\Delta X_t = \alpha\beta_t' X_{t-1} + \varepsilon_t. \tag{41}$$

Now assume that the time-varying cointegration parameters are stationary, e.g. so that it can be represented as an AR(1) process with unconditional mean β:

$$\beta_t = (1 - \rho_\beta)\,\beta + \rho\beta_{t-1} + \varepsilon_{\beta,t}, \tag{42}$$

where ρ_β is an autoregressive parameter and $\varepsilon_{\beta,t}$ is an IID error term with variance σ_β^2. The time-varying parameter can be rewritten as:

$$\beta_t = \beta + \rho\,(\beta_{t-1} - \beta) + \varepsilon_{\beta,t}, \tag{43}$$

and using this specification the cointegrated system can be written as:

$$\Delta X_t = \alpha\beta' X_{t-1} + \alpha\,(\beta_t - \beta)'\,X_{t-1} + \varepsilon_t. \tag{44}$$

As the term $(\beta_t - \beta)$ is a mean zero stationary process the product $(\beta_t - \beta)'\,X_{t-1}$ becomes 'stochastically trendless', and the system becomes 'stochastically cointegrated', see McCabe et al. (2003). Based on simulations, Tabor (2013) shows that the CVAR model gives a consistent estimate of the unconditional mean of the time-varying cointegration parameter in a bivariate system, i.e. a consistent estimate of β. This is possible because the lag structure of the CVAR model can capture the persistence in the 'stochastically trendless' term. However, if there is a high degree of persistence in the time-varying parameter—i.e. ρ_β is smaller than, but close to 1—the underlying parameter instability shows up as an additional degree of persistence in the estimated CVAR model, which causes the estimated adjustment coefficients to be biased towards 0.

5 Estimation Results

For each of the simulated time-series of the stock price p_t and current earnings x_t a CVAR model is estimated based on an automated procedure. The automatic procedure first selects a lag-length k for the unrestricted VAR model in equation (38), where the lag-length is selected as the lowest number where the test of no autocorrelation cannot be rejected with a p-value of 0.05. Given the lag-length, the automated procedure tests for univariate and multivariate autocorrelation, normality, and ARCH in the estimated residuals. Next, the rank test for reduced rank is performed and the largest roots of the companion matrix are calculated. Finally, the automatic procedure estimates the reduced rank CVAR model with a rank of $r = 1$ imposed irrespective of the conclusion of the rank test. For details on the estimation and testing procedures see Johansen (1996) and Juselius (2006), and references therein.

Tables 1-9 in the appendix show the average results over the $S = 1.000$ replications for each of the simulated series and estimations, and for the three different sample lengths considered. It should be noted that the samples of $T = 1.000$ observations are included with the purpose to show the asymptotic results based on a long sample.

Table 1 shows the average simulated probabilities of a break in the segmented trend or a non-moderate revision of the forecasting strategies for each of the specification, as well as the number of breaks occuring in each of the two per 100 observations. It can be seen that as the range for drawing non-moderate revisions increases, the average simulated probability of a non-moderate revision increases from 2 to 3 percent. Hence the number of simulated non-moderate revisions increases from 2.0 to 3.4. As the threshold parameter c_μ used in the logistic function to simulate the probabilities of a reversal in the trend increases, the average simulated probability of a reversal decreases almost exponentially from 8 to 2 percent as c_μ falls from 2.0 to 4.0. As the threshold value increases, fewer simulated reversals in the segmented trend occur. For $c_\mu = 2.0$ an average of 8 reversals occur per 100 observations, meaning that the swings in earnings around the long-run trend are not very long and persistent. However, for $c_\mu = 4.0$ the number of simulated reversals decreses to 1.6 per 100 observations, which implies that the movements in earnings away from the long-run trend are very long and persistent.

Table 2 shows the chosen lag-lengths. From the table it can be seen that as the threshold value for the breaks in the segmented trend, c_μ^i, increases, the number of lags needed in the unrestricted model to be able to not reject no autocorrelation increases. The same holds for an increase in the range for non-moderate revisions, although the effect appears to be smaller. Furthermore, it can be seen that the number of lags needed in the model increases with the sample size.

Table 3 shows the misspecification tests for no autocorrelation in the estimated residuals. It is of interest that it is possible to get non-autocorrelated residuals by choosing an apropriate lag-length in all cases. This is important for inference in the CVAR model, as autocorrelation in the residuals renders basically all inference invalid, and it shows that the flexibility of the lag structure can capture the persistent deviations from the estimated long-run structure caused by the underlying bounded instability.

The misspecification tests for normality of the estimated residuals are presented in Table 4. The results show that only with low values for both τ_b and c_μ, combined with a sample of $T = 200$, can normality of the residuals not be rejected based on the multivariate test, and even in these cases the results are really borderline. In all other cases normality is rejected based on the multivariate test. However, in most cases univariate normality of the residuals in the equation for the stock price cannot be rejected for samples of $T = 200$. By contrast, univariate normality of the residuals in the equation for current earnings cannot be rejected for all specifications and sample lengths.

By looking at Table 5, which shows the univariate skewness and excess kurtosis of the standardized estimated residuals[6], it can be concluded that the rejection of univariate normality in the stock price equation and the rejection of multivariate normality is caused by a very large degree of excess kurtosis. Thus, the densities of the residuals have 'fat tails', which appears to primarily associated with a few large outliers due to large non-moderate revisions in b_t. These outliers can easily be spotted based on a graphical inspection of the data—see for example the illustration above—and we argue that a careful empirical analysis and modeling would—and should—capture the outliers by including a few dummy variables in the model. Though, it is worth mentioning that the CVAR is quite robust to excess kurtosis, see Juselius (2006). By contrast, skewness is more problematic for inference in the CVAR model, but the results in Table 5 shows that skewness is not a problem.

Table 6 shows the final misspecification test, and the results show that no ARCH cannot be rejected for all specifications with high p-values. This is not surprising as the variance of the random shocks was assumed constant in the simulations. However, ARCH-effects in the residuals might also arise from time-varying parameters, but there do not appear to be any noticable volatility clustering in the residuals.

The reduced rank tests are reported in Table 7. The reduced rank tests test the model with a rank of $r = 0, 1$, respectively, against the unrestricted model with full rank $r = p$. A rank of $r = 0$ corresponds to no cointegration in the system, while a rank of $r = 1$ corresponds to one cointegration relation and $p - r = 1$ common stochastic trend in the system. For all specifications a rank of $r = 1$ cannot be rejected on average over all repititions, with p-values well over 0.05 in most cases. However, only for low values of τ_b and c_μ can a rank of $r = 0$ be rejected, which would lead us to choose a reduced

[6]The skewness of the standardized normally distributed residuals should be 0.0 , and the excess kurtosis 3.0.

rank of $r = 1$. It must be pointed out, though, that in general setting the rank is a difficult choice which should not be based solely on the trace test, but by a combination of different indices—such as the number of near-unit roots in the system—as suggested by Juselius (2006).

It is clear from Table 7, that increasing τ_b does not appear to have a large impact on the rank tests, while an increase in c_μ has a large impact leading the test for a rank of $r = 0$ to not be rejected, so that the rank test indicates a preferred rank of $r = 0$. This indicates that the larger fluctuations of current earnings around the long-run trend, the harder it is to find cointegration between the stock price and earnings based on the multivariate rank test. This result is not surprising as an increase in τ_μ makes the deviations between current earnings and the long-run trend in earnings longer and more persistent as the segmented trend is allowed to move further away from $\overline{x}_t$ before a reversal occurs. The greater persistence in the relation between the current earnings and the long-run trend in earnings implies that deviations from the common stochastic trend in the estimated CVAR model becomes more persistent. The greater persistence implies a second near-unit root in the system, which can be seen from the columns with the roots of the companion matrix in Table 7, and simultaneously that the estimated cointegration adjustment coefficients $\widehat{\alpha}_i$ for $i = p, x$ in the reduced rank model with $r = 1$ decrease, as the estimated adjustment to the cointegration relations becomes slower, see Table 9.

Tables 8 and 9 present the estimated cointegration coefficients $\widehat{\beta}$ and the adjustment coefficients $\widehat{\alpha}$. The cointegration relations are normalized on $\widehat{\beta}_1$, which is the coefficient to the stock price, so the cointegration relations are given by $p_t - \widehat{\beta}_2 x_t$. Hence, the normalized coefficients are not shown in Table 8. Moreover, Table 8 presents the average estimates of $\widehat{\beta}_2$ over all $S = 1.000$ replications, as well as an average over all replications excluding a total of 18 out of 48.000 very influential estimated, where the estimated coefficient $\left|\widehat{\beta}_2\right| > 1.000$. For transparency, the average estimates with and without the 18 very influential estimates are shown, but standard errors and t-values are only shown for the latter. Finally, Table 8 reports the averages over all replications of the individual sample averages of b_t, as well as the average difference between the estimate $\widehat{\beta}_2$ and the sample average b_t.

The results show that the estimated coefficients are fairly close to the sample averages of b_t (which are very close to B as expected) when the sample size is long. For a sample size of $T = 200$ the estimated coefficients are in many cases far from the sample averages of b_t. Though, we must point out that even after excluding the 18 most influential replications, the averages of the estimated coefficients are still very influenced by a few number of extreme estimates, which a careful econometric analysis would not get.

From Table 9 it can be seen that the estimated adjustment coefficients are found to be equilibrium adjusting in all specifications (i.e. $\widehat{\alpha}_2 > 0$), while the stock price is generally found to be equilibrium-increasing for low values of τ_b and c_μ (i.e. $\widehat{\alpha}_1 > 0$), and equilibrium-adjusting for higher values of τ_b and c_μ (i.e. $\widehat{\alpha}_1 < 0$). Moreover, in the former case the estimated adjustment coefficients are on average significant, but as τ_b and c_μ increase the significance decreases, and eventually both adjustment coefficients become insignificant. As mentioned above, these results can be understood from the fact that increasing τ_b and c_μ allows for a greater degree of persistence in the fluctuations of p_t and x_t around the common long-run trend in earnings.

6 Conclusion

To conclude on the simulations, the results from the automated estimations indicate that the CVAR model—with its system approach, lag structure, and decomposition of the data according to its degree of persistence—can be used as a surprisingly good statistical representation of the simulated data with an adequate lag structure. Importantly, the estimations also show that in many cases a 'correct' reduced rank of $r = 1$ is found, and the estimated cointegration coefficients are close to the corresponding parameters in the simulations. Finally, we find a large degree of persistence in the system, which indicates that the underlying bounded instability in the individual processes and parameters shows up as persistence in the CVAR model.

The results are surprising, in particular when one takes into account that the specification of the simulations do not correspond to the specification of the CVAR model, and that the simulations have bounded instability in the parameters, while the CVAR model has constant parameters.

It is important to note that the inclusion of lagged first-differences in the CVAR model appears to be an extremely important element in the specification of a general unrestricted VAR model as a statistically valid representation of the data. It appears that the underlying bounded instability in the stochastic processes and parameters can be fairly well captured by the lagged first-differences in the short-run structure—so that the estimated residuals are fairly well-specified—while the cointegration relations capture the stable long-run relations in the data.

However, it is worth pointing out that the simulations were based on bounded instability in the short run, but with stability in the causal structure in the long run. On that basis the results here might not be very surprising, and it will be interesting to get new results we expand the fairly simple simulations considered here. In our future work on bridging IKE models with the CVAR model and more generally the 'data-first approach' to econometrics, we will allow for more variables to enter the forecasting strategies in the simulations, we will allow for contingent change that is not bounded within a narrow range, and we will focus directly on testing for structural change in the CVAR model.

A Graphs of Simulated Series and Results from CVAR Estimations

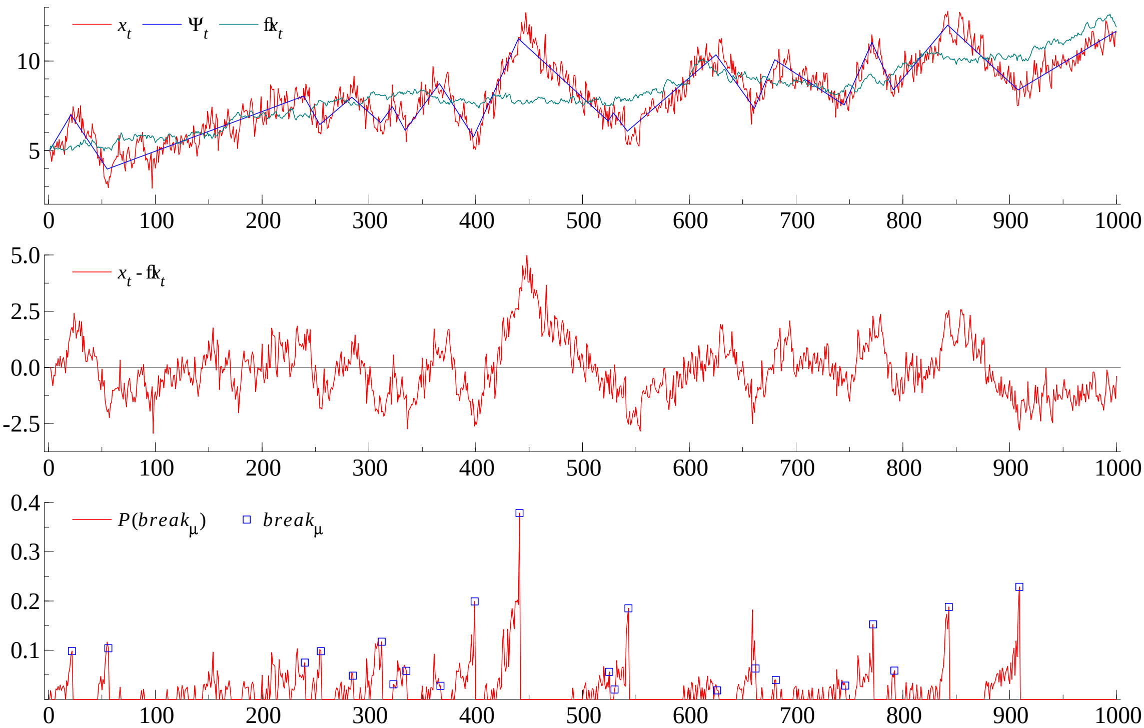


Figure 1: Current earnings, the segmented trend, and the long-run trend in earnings. The upper panel shows the long-run trend in earnings $\bar{x}_t$ (green line), with the segmented trend Ψ_t (blue line) and current earnings x_t (red line). The middle panel shows the deviation between current earnings and their long-run trend, $\bar{x}_t - x_t$, which determines the simulated probability of a break in the segmented trend. The simulated probabilities are shown in the lower panel, where the blue squares indicate a reversal in the direction of the segmented trend.

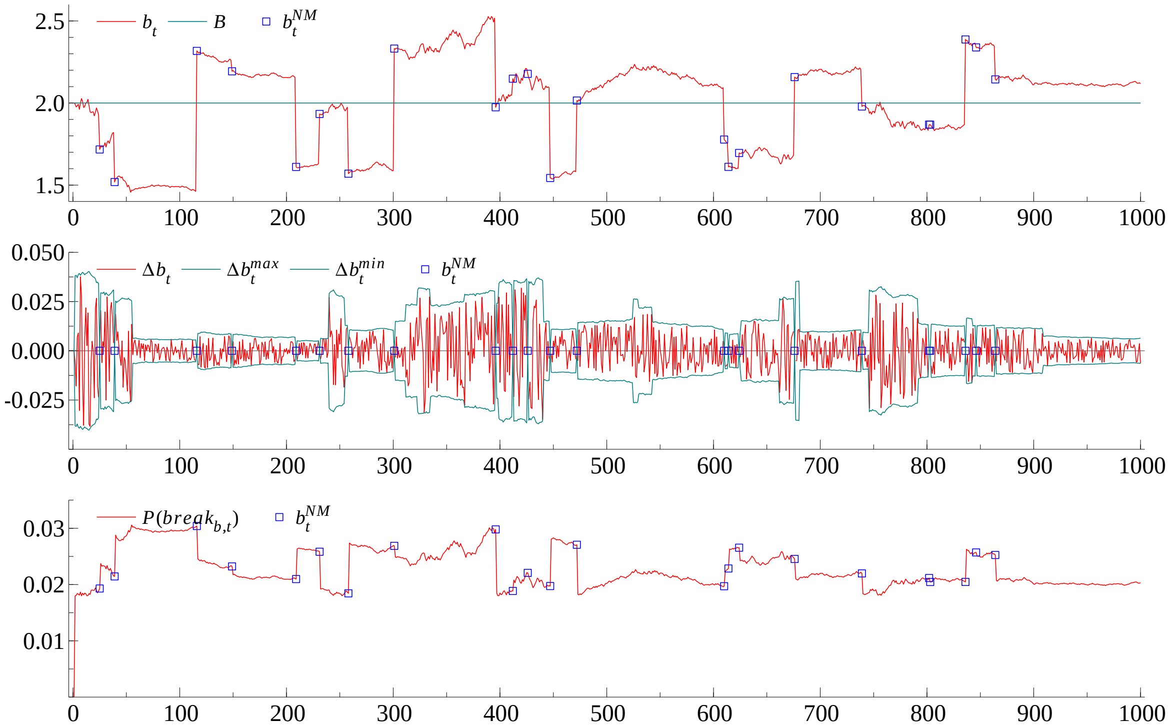


Figure 2: Revision of forecasting strategies. The upper panel shows the weights attached to current earnings in the forecasting strategies over time (red line) along with non-moderate revisions the forecasting strategies (blue squares) and the parameter B (green line). The middle panel shows the qualitative ranges imposed on the revisions of the forecasting strategies by the Guardedly Moderate Revisions condition (green lines), the simulated revisions of the forecasting strategies (red line), and the non-moderate revisions the forecasting strategies (blue squares). The lower panel shows the simulated probability of a non-moderate revision (red line) and the non-moderate revisions the forecasting strategies (blue squares).

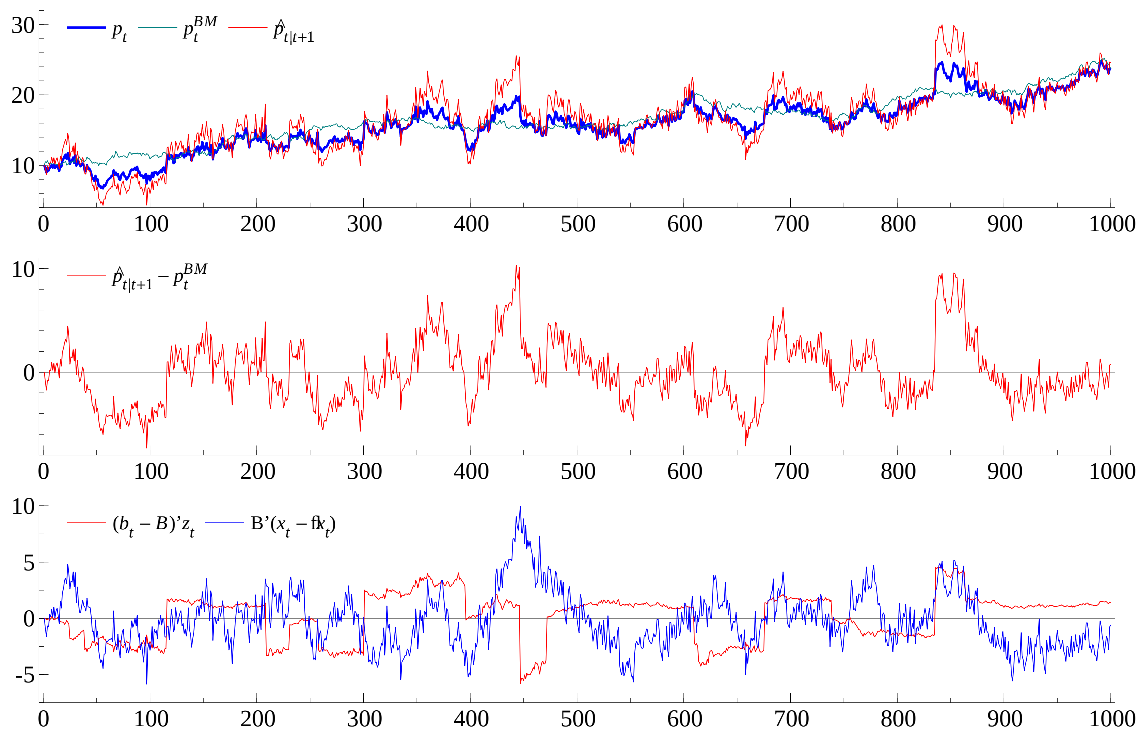
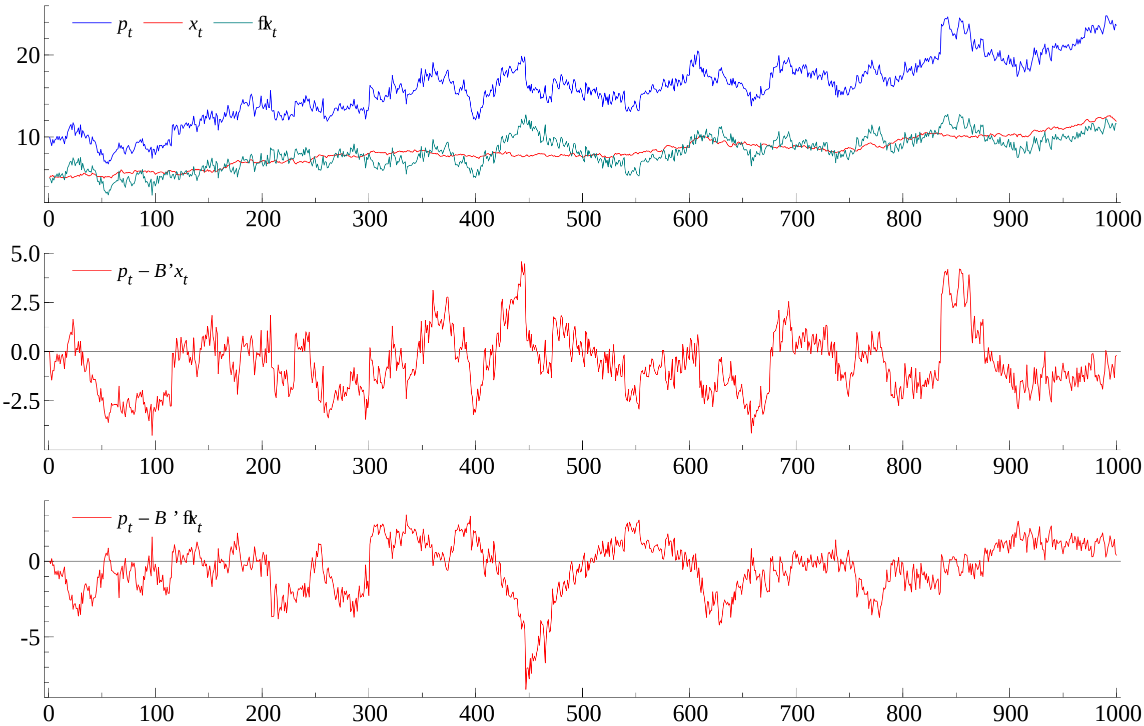


Figure 3: The stock price, benchmark price, price forecast. The upper panel shows the simulated stock price (blue line), the benchmark price (green line), and the price forecast (red line). The middle panel shows the deviation between the price forecast and the benchmark price. The lower panel shows the decomposition of the deviation between the price forecast and the benchmark price into two terms, which are individually bounded: $(b_t - B)'z_t$ and $\mathrm{B}'(x_t - \bar{x}_t)$.



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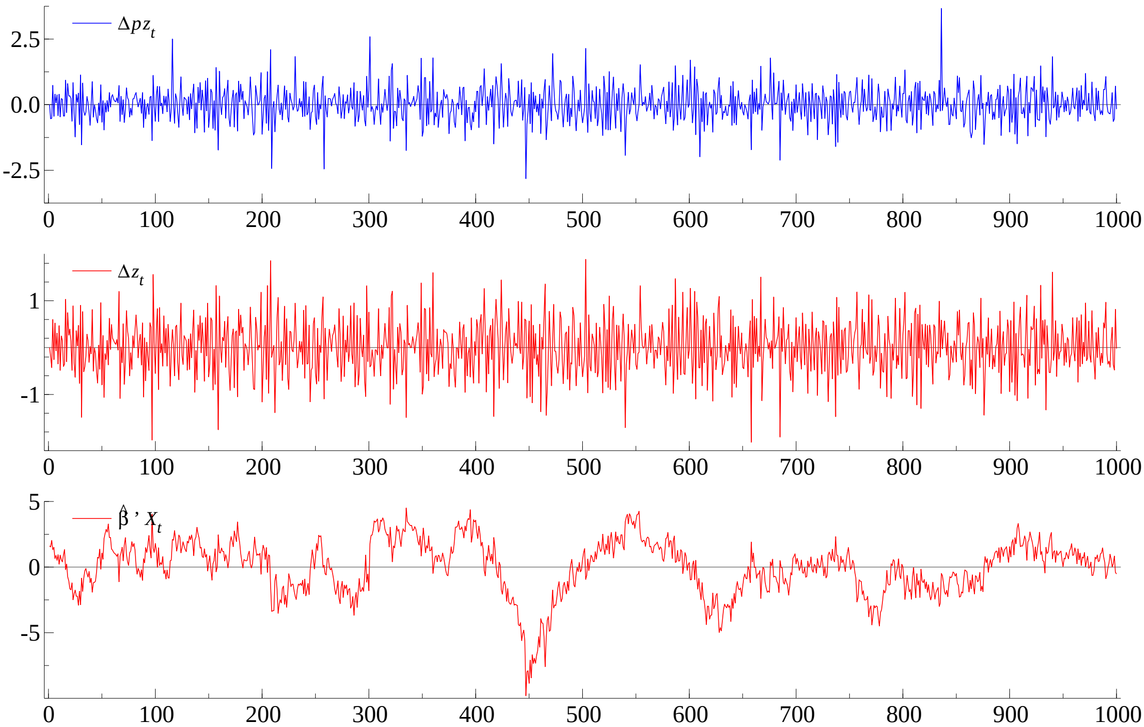

Figure 5: Estimated cointegration relation and first-differences. The upper and middle panels show the first-differences of the stock price and earnings data, which are analyzed econometrically with a CVAR model. The lower panel shows the estimated cointegration relation $\hat{\beta}' X_t$.

Table 1: Breakpoints in the Simulated Data

i	τ_b^i	c_μ^i	T	$P(break_b)$[a]	$breaks_b$[b]	$P(break_\mu)$[a]	$breaks_\mu$[b]
1	0.25	2.00	200	0.02	2.0	0.08	7.9
			400	0.02	2.0	0.08	7.7
			1000	0.02	2.1	0.08	7.6
2	0.25	3.00	200	0.02	2.0	0.04	4.4
			400	0.02	2.0	0.04	4.2
			1000	0.02	2.1	0.04	4.2
3	0.25	4.00	200	0.02	2.0	0.03	2.6
			400	0.02	2.0	0.03	2.5
			1000	0.02	2.1	0.02	2.5
4	0.25	5.00	200	0.02	2.0	0.02	1.7
			400	0.02	2.0	0.02	1.7
			1000	0.02	2.1	0.02	1.6
5	0.50	2.00	200	0.02	2.2	0.08	7.9
			400	0.02	2.2	0.08	7.7
			1000	0.02	2.3	0.08	7.6
6	0.50	3.00	200	0.02	2.2	0.04	4.4
			400	0.02	2.2	0.04	4.2
			1000	0.02	2.3	0.04	4.2
7	0.50	4.00	200	0.02	2.2	0.03	2.6
			400	0.02	2.2	0.03	2.5
			1000	0.02	2.3	0.02	2.5
8	0.50	5.00	200	0.02	2.2	0.02	1.7
			400	0.02	2.2	0.02	1.7
			1000	0.02	2.3	0.02	1.6
9	1.00	2.00	200	0.03	2.6	0.08	7.9
			400	0.03	2.7	0.08	7.7
			1000	0.03	2.8	0.08	7.6
10	1.00	3.00	200	0.03	2.6	0.04	4.4
			400	0.03	2.7	0.04	4.2
			1000	0.03	2.8	0.04	4.2
11	1.00	4.00	200	0.03	2.6	0.03	2.6
			400	0.03	2.7	0.03	2.5
			1000	0.03	2.8	0.02	2.5
12	1.00	5.00	200	0.03	2.6	0.02	1.7
			400	0.03	2.7	0.02	1.7
			1000	0.03	2.8	0.02	1.6
13	1.50	2.00	200	0.03	3.1	0.08	7.9
			400	0.03	3.2	0.08	7.7
			1000	0.03	3.4	0.08	7.6
14	1.50	3.00	200	0.03	3.1	0.04	4.4
			400	0.03	3.2	0.04	4.2
			1000	0.03	3.4	0.04	4.2
15	1.50	4.00	200	0.03	3.1	0.03	2.6
			400	0.03	3.3	0.03	2.5
			1000	0.03	3.4	0.02	2.5
16	1.50	5.00	200	0.03	3.1	0.02	1.7
			400	0.03	3.2	0.02	1.7
			1000	0.03	3.4	0.02	1.6

All reported values are averages over $S = 1000$ replications.

[a] Average simulated probability of a non-moderate revision of the forecasting strategies and a reversal in the segmented trend, respectively.

[b] Average number of breakpoints per 100 observations.

Table 2: Selected Lag-Lengths in the Unrestricted Model

i	τ_b^i	c_μ^i	T	$Av(k)$[a]	$k=1$[b]	$k=2$[b]	$k=3$[b]	$k\geq 4$[b]
1	0.25	2.00	200	1.18	0.84	0.15	0.01	0.00
			400	1.67	0.49	0.38	0.11	0.02
			1000	3.43	0.02	0.18	0.38	0.42
2	0.25	3.00	200	1.33	0.71	0.25	0.04	0.00
			400	2.01	0.29	0.46	0.22	0.04
			1000	3.63	0.00	0.10	0.37	0.53
3	0.25	4.00	200	1.50	0.58	0.35	0.07	0.00
			400	2.38	0.15	0.44	0.32	0.09
			1000	3.90	0.00	0.05	0.30	0.65
4	0.25	5.00	200	1.64	0.49	0.39	0.10	0.02
			400	2.65	0.08	0.38	0.39	0.15
			1000	4.09	0.00	0.03	0.24	0.73
5	0.50	2.00	200	1.22	0.80	0.18	0.02	0.00
			400	1.85	0.39	0.42	0.16	0.03
			1000	3.99	0.00	0.07	0.27	0.66
6	0.50	3.00	200	1.36	0.68	0.28	0.04	0.00
			400	2.14	0.25	0.44	0.25	0.06
			1000	3.96	0.00	0.05	0.28	0.68
7	0.50	4.00	200	1.50	0.57	0.36	0.06	0.00
			400	2.46	0.14	0.40	0.34	0.12
			1000	4.04	0.00	0.03	0.26	0.71
8	0.50	5.00	200	1.64	0.48	0.40	0.10	0.01
			400	2.66	0.07	0.38	0.39	0.16
			1000	4.18	0.00	0.02	0.21	0.76
9	1.00	2.00	200	1.27	0.77	0.20	0.03	0.00
			400	2.09	0.29	0.40	0.24	0.06
			1000	4.38	0.00	0.04	0.19	0.78
10	1.00	3.00	200	1.39	0.66	0.29	0.04	0.01
			400	2.32	0.19	0.40	0.30	0.10
			1000	4.16	0.00	0.03	0.22	0.75
11	1.00	4.00	200	1.54	0.55	0.36	0.08	0.01
			400	2.56	0.12	0.36	0.38	0.14
			1000	4.23	0.00	0.02	0.20	0.78
12	1.00	5.00	200	1.65	0.48	0.40	0.10	0.01
			400	2.72	0.07	0.36	0.39	0.18
			1000	4.31	0.00	0.01	0.19	0.80
13	1.50	2.00	200	1.32	0.74	0.21	0.05	0.00
			400	2.22	0.25	0.41	0.25	0.10
			1000	4.50	0.00	0.03	0.17	0.81
14	1.50	3.00	200	1.43	0.65	0.28	0.06	0.01
			400	2.42	0.16	0.40	0.32	0.12
			1000	4.32	0.00	0.02	0.18	0.80
15	1.50	4.00	200	1.59	0.53	0.36	0.10	0.01
			400	2.58	0.11	0.38	0.36	0.15
			1000	4.31	0.00	0.01	0.18	0.81
16	1.50	5.00	200	1.70	0.46	0.40	0.12	0.02
			400	2.76	0.07	0.34	0.39	0.20
			1000	4.39	0.00	0.01	0.16	0.82

All reported values are averages over $S = 1000$ replications.
[a] Average lag-length k.
[b] Percentage with lag length $k = 1, 2, 3$ and $k \geq 4$, respectively.

Table 3: Misspecification Tests Part 1: Autocorrelation

				Vector test no autocorr. order 1[a]		Vector test no autocorr. order 1-2[b]		Univar. test no autocorr. order 1 in $\hat{\varepsilon}_{1t}$[b]		Univ. test no autocorr. order 1 in $\hat{\varepsilon}_{2t}$[b]	
i	τ_b^i	c_μ^i	T	$\chi^2(4)$	$p-val$	$\chi^2(8)$	$p-val$	$\chi^2(4)$	$p-val$	$\chi^2(4)$	$p-val$
1	0.25	2.00	200	4.45	0.43	9.61	0.79	4.00	0.66	3.91	0.67
			400	5.37	0.32	12.13	0.67	6.31	0.45	6.52	0.46
			1000	5.77	0.29	13.79	0.41	7.71	0.16	8.23	0.15
2	0.25	3.00	200	4.93	0.37	10.69	0.76	4.96	0.59	5.18	0.60
			400	5.61	0.30	12.71	0.60	6.78	0.32	7.25	0.31
			1000	5.64	0.30	12.76	0.45	6.94	0.21	7.50	0.20
3	0.25	4.00	200	5.16	0.34	11.53	0.71	5.71	0.48	6.26	0.48
			400	5.61	0.30	12.79	0.54	6.69	0.26	7.44	0.24
			1000	5.57	0.31	12.53	0.46	6.72	0.23	7.39	0.21
4	0.25	5.00	200	5.40	0.32	12.14	0.66	6.10	0.42	6.80	0.39
			400	5.53	0.32	12.91	0.50	6.54	0.24	7.40	0.22
			1000	5.50	0.32	12.34	0.47	6.79	0.24	7.35	0.22
5	0.50	2.00	200	4.55	0.42	9.82	0.79	4.00	0.63	4.24	0.64
			400	5.49	0.31	12.54	0.62	6.12	0.38	7.17	0.37
			1000	5.84	0.28	14.28	0.36	7.28	0.17	8.88	0.13
6	0.50	3.00	200	4.91	0.37	10.81	0.75	4.70	0.58	5.35	0.58
			400	5.54	0.31	12.82	0.57	6.29	0.32	7.49	0.29
			1000	5.54	0.31	12.90	0.43	6.33	0.24	7.69	0.19
7	0.50	4.00	200	5.24	0.34	11.71	0.70	5.31	0.49	6.40	0.46
			400	5.47	0.32	12.66	0.54	6.06	0.29	7.43	0.24
			1000	5.60	0.31	12.45	0.46	6.00	0.27	7.44	0.20
8	0.50	5.00	200	5.32	0.33	12.14	0.65	5.60	0.44	6.86	0.38
			400	5.50	0.32	12.75	0.50	5.86	0.28	7.48	0.21
			1000	5.42	0.33	12.36	0.47	5.89	0.28	7.39	0.22
9	1.00	2.00	200	4.55	0.41	10.31	0.77	3.38	0.64	4.58	0.62
			400	5.63	0.30	13.19	0.55	4.66	0.45	7.57	0.29
			1000	5.76	0.28	14.90	0.34	4.92	0.34	9.13	0.12
10	1.00	3.00	200	4.93	0.37	11.12	0.74	3.71	0.61	5.63	0.55
			400	5.58	0.31	12.87	0.53	4.39	0.43	7.54	0.26
			1000	5.68	0.30	13.44	0.41	4.58	0.37	8.03	0.18
11	1.00	4.00	200	5.18	0.34	11.70	0.70	4.13	0.56	6.38	0.46
			400	5.46	0.32	12.72	0.51	4.50	0.41	7.51	0.23
			1000	5.47	0.32	12.80	0.46	4.06	0.42	7.41	0.22
12	1.00	5.00	200	5.41	0.32	12.32	0.65	4.44	0.51	7.00	0.37
			400	5.45	0.32	12.72	0.50	4.36	0.41	7.44	0.22
			1000	5.36	0.33	12.43	0.48	3.95	0.44	7.17	0.24
13	1.50	2.00	200	4.61	0.41	10.47	0.75	2.80	0.66	4.68	0.62
			400	5.55	0.31	13.35	0.53	3.35	0.55	7.68	0.27
			1000	5.94	0.27	15.25	0.35	3.38	0.50	8.99	0.13
14	1.50	3.00	200	4.99	0.36	11.27	0.73	3.04	0.64	5.66	0.55
			400	5.44	0.32	12.97	0.52	3.26	0.55	7.48	0.24
			1000	5.72	0.30	13.58	0.43	3.08	0.54	7.63	0.20
15	1.50	4.00	200	5.15	0.34	11.71	0.69	3.24	0.62	6.35	0.46
			400	5.46	0.32	12.95	0.50	3.25	0.55	7.47	0.23
			1000	5.66	0.31	13.32	0.46	3.07	0.56	7.20	0.22
16	1.50	5.00	200	5.40	0.32	12.37	0.64	3.48	0.59	6.84	0.37
			400	5.41	0.32	12.81	0.49	3.20	0.54	7.40	0.22
			1000	5.49	0.32	12.86	0.47	2.95	0.56	6.83	0.25

All reported values are averages over $S = 1000$ replications.

[a] Multivariate test for no autocorrelation of order 1 and order $1-2$, respectively, in the estimated residuals. The first columns report the test values, while the second report the corresponding p-values.

[b] Univariate tests for no autocorrelation of order 1 in the estimated residuals. The first columns report the test values, while the second report the corresponding p-values.

Table 4: Misspecification Tests Part 2: Normality

i	τ_b^i	c_μ^i	T	Vector test normality[b] $\chi^2(4)$	$p-val$	Univar. test normality of $\hat{\varepsilon}_{1t}$[b] $\chi^2(2)$	$p-val$	Univar. test normality of $\hat{\varepsilon}_{2t}$[b] $\chi^2(2)$	$p-val$
1	0.25	2.00	200	81.75	0.06	2.79	0.46	1.97	0.51
			400	335.01	0.01	5.48	0.37	1.93	0.51
			1000	2735.45	0.00	48.69	0.09	1.91	0.51
2	0.25	3.00	200	83.96	0.06	2.75	0.47	1.98	0.51
			400	334.08	0.01	5.13	0.39	2.02	0.51
			1000	2772.08	0.00	47.36	0.09	1.92	0.51
3	0.25	4.00	200	82.29	0.07	2.68	0.47	1.94	0.52
			400	334.07	0.01	5.22	0.39	1.88	0.52
			1000	2758.95	0.00	46.09	0.10	1.95	0.51
4	0.25	5.00	200	85.63	0.07	2.76	0.47	1.92	0.52
			400	339.41	0.01	5.72	0.39	1.83	0.53
			1000	2788.76	0.00	47.03	0.11	1.95	0.51
5	0.50	2.00	200	207.18	0.02	12.61	0.28	1.97	0.51
			400	842.87	0.00	54.35	0.12	1.96	0.50
			1000	5069.20	0.00	694.32	0.00	1.90	0.52
6	0.50	3.00	200	202.24	0.03	12.04	0.29	1.96	0.51
			400	845.85	0.00	52.95	0.12	2.02	0.50
			1000	5125.59	0.00	710.91	0.00	1.93	0.51
7	0.50	4.00	200	203.07	0.03	12.06	0.30	1.95	0.52
			400	839.81	0.00	52.80	0.13	1.91	0.52
			1000	5169.32	0.00	717.68	0.00	1.95	0.51
8	0.50	5.00	200	205.12	0.03	13.47	0.30	1.92	0.52
			400	829.39	0.00	53.86	0.13	1.84	0.52
			1000	5147.33	0.00	718.60	0.00	1.94	0.51
9	1.00	2.00	200	389.33	0.01	89.53	0.08	1.97	0.51
			400	1378.13	0.00	411.27	0.01	1.97	0.50
			1000	6564.38	0.00	3272.08	0.00	1.90	0.52
10	1.00	3.00	200	382.50	0.01	87.53	0.08	1.97	0.51
			400	1396.18	0.00	412.64	0.01	2.03	0.50
			1000	6627.71	0.00	3315.64	0.00	1.92	0.51
11	1.00	4.00	200	378.77	0.01	90.11	0.09	1.93	0.52
			400	1383.35	0.00	413.79	0.01	1.93	0.51
			1000	6725.62	0.00	3380.56	0.00	1.94	0.51
12	1.00	5.00	200	391.53	0.01	93.19	0.09	1.92	0.52
			400	1362.78	0.00	421.65	0.01	1.87	0.52
			1000	6727.86	0.00	3406.74	0.00	1.94	0.51
13	1.50	2.00	200	496.41	0.01	206.98	0.03	1.99	0.51
			400	1562.25	0.00	811.22	0.00	1.97	0.51
			1000	6516.40	0.00	4707.44	0.00	1.88	0.52
14	1.50	3.00	200	495.58	0.01	205.26	0.04	1.98	0.51
			400	1564.72	0.00	811.76	0.00	2.01	0.50
			1000	6598.74	0.00	4783.97	0.00	1.91	0.51
15	1.50	4.00	200	483.63	0.01	200.94	0.04	1.93	0.52
			400	1563.63	0.00	816.23	0.00	1.95	0.51
			1000	6616.48	0.00	4812.31	0.00	1.93	0.51
16	1.50	5.00	200	495.95	0.01	206.62	0.04	1.91	0.52
			400	1558.25	0.00	824.87	0.00	1.88	0.52
			1000	6667.47	0.00	4858.83	0.00	1.92	0.52

All reported values are averages over $S = 1000$ replications.
[a] Multivariate test for normality of the estimated residuals.
[b] Univariate tests for normality of the estimated residuals.

Table 5: Misspecification Tests Part 3: Skewness, Kurtosis, and Standard Deviation

i	τ_b^i	c_μ^i	T	Skewness[a] $\hat{\varepsilon}_{1t}$	Skewness[a] $\hat{\varepsilon}_{2t}$	Kurtosis[b] $\hat{\varepsilon}_{1t}$	Kurtosis[b] $\hat{\varepsilon}_{2t}$	Std.dev.[c] $\hat{\varepsilon}_{1t}$	Std.dev.[c] $\hat{\varepsilon}_{2t}$
1	0.25	2.00	200	-0.01	0.00	3.12	2.98	0.56	0.54
			400	-0.02	0.00	3.28	2.97	0.57	0.55
			1000	-0.03	-0.00	4.03	3.00	0.59	0.55
2	0.25	3.00	200	0.00	0.00	3.11	2.97	0.57	0.55
			400	-0.01	0.00	3.26	2.98	0.58	0.55
			1000	-0.02	-0.00	4.01	3.00	0.59	0.55
3	0.25	4.00	200	0.01	0.00	3.11	2.97	0.57	0.55
			400	-0.00	-0.00	3.25	2.97	0.58	0.55
			1000	-0.01	-0.00	3.97	3.00	0.59	0.55
4	0.25	5.00	200	0.01	0.00	3.12	2.97	0.58	0.56
			400	-0.00	-0.00	3.28	2.97	0.58	0.55
			1000	-0.01	-0.00	3.98	3.00	0.59	0.55
5	0.50	2.00	200	0.01	0.00	3.96	2.98	0.59	0.55
			400	-0.01	0.00	5.04	2.97	0.61	0.55
			1000	-0.00	-0.00	10.69	3.00	0.66	0.55
6	0.50	3.00	200	0.02	-0.00	3.92	2.97	0.59	0.55
			400	-0.01	-0.00	5.02	2.97	0.61	0.55
			1000	0.00	-0.00	10.88	3.00	0.66	0.55
7	0.50	4.00	200	0.04	0.00	3.91	2.97	0.60	0.55
			400	0.01	-0.00	5.02	2.97	0.61	0.55
			1000	0.00	-0.00	10.95	3.00	0.65	0.55
8	0.50	5.00	200	0.05	0.00	4.02	2.98	0.60	0.56
			400	0.01	-0.00	5.05	2.97	0.61	0.55
			1000	0.00	-0.00	11.05	3.00	0.65	0.55
9	1.00	2.00	200	0.10	0.00	8.97	2.98	0.68	0.55
			400	0.09	-0.00	14.95	2.97	0.74	0.56
			1000	0.06	-0.01	35.15	2.99	0.91	0.56
10	1.00	3.00	200	0.12	0.00	8.90	2.97	0.68	0.55
			400	0.08	-0.00	15.06	2.97	0.74	0.55
			1000	0.06	-0.00	35.64	2.99	0.91	0.55
11	1.00	4.00	200	0.13	0.00	9.03	2.97	0.69	0.55
			400	0.11	-0.00	15.25	2.97	0.74	0.55
			1000	0.03	-0.00	36.42	3.00	0.91	0.55
12	1.00	5.00	200	0.12	0.00	9.33	2.98	0.69	0.56
			400	0.08	-0.00	15.78	2.97	0.74	0.55
			1000	0.04	-0.00	36.92	3.00	0.91	0.55
13	1.50	2.00	200	0.10	-0.00	15.89	2.98	0.83	0.55
			400	0.11	-0.00	26.29	2.97	0.95	0.56
			1000	0.09	-0.01	49.17	2.99	1.29	0.56
14	1.50	3.00	200	0.11	0.00	16.03	2.97	0.84	0.55
			400	0.11	-0.00	26.38	2.97	0.95	0.55
			1000	0.10	-0.00	49.69	2.99	1.29	0.55
15	1.50	4.00	200	0.12	0.00	16.08	2.97	0.84	0.55
			400	0.15	-0.00	26.78	2.97	0.95	0.55
			1000	0.08	-0.00	50.28	2.99	1.29	0.55
16	1.50	5.00	200	0.12	0.00	16.52	2.98	0.85	0.56
			400	0.08	-0.00	27.28	2.97	0.96	0.55
			1000	0.13	-0.00	51.53	2.99	1.29	0.55

All reported values are averages over $S = 1000$ replications.

[a] The skewness of the estimated residuals is calculated as $skewness_i = \sqrt{T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_{it}^3}$, where $\hat{\varepsilon}_{it}$ are the estimated system residuals for $i = 1, 2$.

[b] The kurtosis of the estimated residuals is calculated as $kurtosis_i = T^{-1}\sum_{t=1}^{T}\hat{\varepsilon}_{it}^4$, where $\hat{\varepsilon}_{it}$ are the estimated system residuals for $i = 1, 2$.

[c] The standard deviation of the estimated residuals $\hat{\varepsilon}_{it}$ for $i = 1, 2$

Table 6: Misspecification Tests Part 4: ARCH

i	τ_b^i	c_μ^i	T	Vector test no. ARCH order 1[a] $\chi^2(9)$	$p-val$	Vector test no ARCH order 1-2[a] $\chi^2(18)$	$p-val$	Univar. test no ARCH order 1 in $\hat{\varepsilon}_{1t}$[b] $\chi^2(1)$	$p-val$	Univ. test no ARCH order 1 in $\hat{\varepsilon}_{2t}$[b] $\chi^2(1)$	$p-val$
1	0.25	2.00	200	31.61	0.38	39.65	0.39	1.02	0.51	0.99	0.51
			400	21.93	0.48	30.60	0.50	0.94	0.53	0.96	0.50
			1000	10.16	0.60	20.28	0.58	1.08	0.51	1.01	0.51
2	0.25	3.00	200	27.75	0.44	36.05	0.45	1.01	0.51	0.96	0.51
			400	17.55	0.55	26.71	0.54	0.95	0.52	0.96	0.50
			1000	9.74	0.61	19.15	0.60	1.10	0.52	1.02	0.51
3	0.25	4.00	200	23.42	0.47	31.56	0.48	1.08	0.49	1.02	0.50
			400	13.77	0.59	22.25	0.59	0.94	0.52	0.99	0.50
			1000	10.11	0.61	19.85	0.60	1.09	0.52	1.01	0.51
4	0.25	5.00	200	20.47	0.51	28.75	0.52	1.06	0.51	0.99	0.51
			400	10.68	0.61	19.53	0.60	0.93	0.52	0.99	0.50
			1000	10.27	0.60	19.78	0.60	1.06	0.51	1.04	0.49
5	0.50	2.00	200	21.06	0.45	30.19	0.45	1.01	0.53	0.99	0.51
			400	15.26	0.51	25.52	0.50	0.97	0.55	0.98	0.50
			1000	11.34	0.58	22.21	0.57	1.17	0.64	1.02	0.51
6	0.50	3.00	200	18.90	0.49	28.24	0.49	0.99	0.53	0.97	0.51
			400	13.24	0.54	23.60	0.52	0.95	0.56	0.97	0.50
			1000	11.31	0.59	22.08	0.57	1.20	0.63	1.01	0.51
7	0.50	4.00	200	16.68	0.50	25.49	0.51	1.05	0.52	1.02	0.49
			400	12.55	0.56	22.42	0.55	0.97	0.55	0.99	0.50
			1000	11.66	0.58	22.53	0.56	1.32	0.63	1.00	0.51
8	0.50	5.00	200	15.38	0.54	24.16	0.55	0.99	0.53	1.00	0.50
			400	11.34	0.56	21.45	0.55	1.06	0.55	0.99	0.50
			1000	11.77	0.58	22.61	0.57	1.26	0.63	1.04	0.49
9	1.00	2.00	200	15.26	0.50	26.19	0.49	0.92	0.61	0.98	0.51
			400	13.55	0.54	25.19	0.52	1.38	0.65	0.97	0.50
			1000	10.79	0.62	21.68	0.59	1.35	0.70	1.04	0.51
10	1.00	3.00	200	14.57	0.52	25.41	0.50	0.96	0.61	0.96	0.51
			400	12.40	0.56	23.86	0.52	1.24	0.65	0.99	0.50
			1000	10.63	0.62	21.23	0.60	1.28	0.70	1.02	0.51
11	1.00	4.00	200	12.69	0.54	22.96	0.52	1.03	0.60	1.00	0.50
			400	12.35	0.56	23.31	0.54	1.20	0.65	1.00	0.50
			1000	10.88	0.62	21.60	0.59	1.48	0.70	1.00	0.51
12	1.00	5.00	200	12.49	0.55	22.50	0.54	0.99	0.61	0.99	0.50
			400	12.51	0.56	23.77	0.53	1.32	0.65	0.99	0.49
			1000	11.17	0.62	21.82	0.60	1.47	0.71	1.04	0.49
13	1.50	2.00	200	14.22	0.53	25.89	0.50	0.98	0.67	0.99	0.51
			400	13.29	0.57	24.82	0.53	2.06	0.68	0.97	0.50
			1000	12.41	0.63	25.99	0.56	2.52	0.65	1.03	0.51
14	1.50	3.00	200	13.88	0.54	25.23	0.51	1.12	0.67	0.98	0.51
			400	11.92	0.60	23.14	0.56	1.88	0.68	0.97	0.50
			1000	12.19	0.62	25.46	0.57	2.37	0.66	1.03	0.51
15	1.50	4.00	200	12.53	0.56	23.17	0.53	1.05	0.66	0.99	0.50
			400	11.92	0.60	22.82	0.57	1.74	0.69	0.98	0.50
			1000	12.34	0.61	25.90	0.56	2.54	0.65	1.03	0.51
16	1.50	5.00	200	11.81	0.57	22.63	0.54	1.02	0.67	1.01	0.49
			400	12.53	0.60	23.61	0.57	2.03	0.69	0.97	0.50
			1000	13.21	0.62	26.23	0.57	2.79	0.66	1.04	0.50

All reported values are averages over $S = 1000$ replications.
[a] Multivariate test for no ARCH of order 1 and order $1-2$, respectively, in the estimated residuals.
[b] Univariate tests for no autocorrelation of order 1 in the estimated residuals.

Table 7: Reduced Rank Determination: Rank Test

				Reduced rank tests $\mathcal{H}(r,s)$[a]				Roots of comp. matrix[b]		
i	τ_b^i	c_μ^i	T	$\mathcal{H}(0,2)$	$p-val$	$\mathcal{H}(1,1)$	$p-val$	$\hat{v}_{1,r=2}$	$\hat{v}_{2,r=2}$	$\hat{v}_{2,r=1}$
1	0.25	2.00	200	31.90	0.01	3.02	0.27	0.97	0.74	0.74
			400	42.55	0.00	2.21	0.32	0.99	0.81	0.81
			1000	48.88	0.00	1.31	0.41	1.00	0.90	0.90
2	0.25	3.00	200	23.16	0.04	2.96	0.26	0.97	0.82	0.82
			400	28.45	0.02	2.18	0.32	0.99	0.88	0.88
			1000	35.12	0.00	1.30	0.41	1.00	0.93	0.93
3	0.25	4.00	200	16.80	0.13	2.66	0.26	0.98	0.88	0.87
			400	18.22	0.09	2.10	0.32	0.99	0.93	0.92
			1000	23.45	0.02	1.28	0.41	1.00	0.96	0.96
4	0.25	5.00	200	12.54	0.27	2.35	0.27	0.98	0.92	0.92
			400	11.96	0.28	1.98	0.32	0.99	0.96	0.96
			1000	15.48	0.10	1.27	0.41	1.00	0.97	0.97
5	0.50	2.00	200	30.11	0.01	3.36	0.21	0.97	0.76	0.76
			400	35.45	0.01	2.97	0.24	0.99	0.84	0.84
			1000	32.71	0.01	1.92	0.29	1.00	0.94	0.94
6	0.50	3.00	200	22.52	0.04	3.26	0.21	0.97	0.83	0.82
			400	25.66	0.03	2.88	0.24	0.99	0.89	0.89
			1000	27.54	0.01	1.89	0.29	1.00	0.95	0.95
7	0.50	4.00	200	16.90	0.12	2.98	0.21	0.97	0.88	0.88
			400	17.85	0.10	2.73	0.24	0.99	0.93	0.93
			1000	21.66	0.03	1.84	0.30	1.00	0.96	0.96
8	0.50	5.00	200	12.97	0.25	2.65	0.22	0.97	0.92	0.91
			400	12.62	0.25	2.50	0.25	0.99	0.96	0.95
			1000	16.38	0.08	1.78	0.30	1.00	0.97	0.97
9	1.00	2.00	200	28.61	0.02	4.19	0.14	0.96	0.78	0.78
			400	29.61	0.03	4.58	0.12	0.98	0.88	0.88
			1000	27.88	0.01	2.98	0.18	0.99	0.95	0.95
10	1.00	3.00	200	22.08	0.05	4.03	0.15	0.96	0.84	0.83
			400	23.61	0.04	4.39	0.13	0.98	0.91	0.91
			1000	26.40	0.01	2.97	0.18	0.99	0.95	0.95
11	1.00	4.00	200	17.29	0.11	3.62	0.15	0.96	0.88	0.87
			400	18.37	0.09	4.09	0.13	0.98	0.93	0.93
			1000	23.76	0.02	2.75	0.19	0.99	0.96	0.96
12	1.00	5.00	200	13.78	0.21	3.12	0.17	0.97	0.91	0.91
			400	14.50	0.17	3.46	0.15	0.98	0.95	0.95
			1000	20.85	0.04	2.54	0.21	0.99	0.96	0.96
13	1.50	2.00	200	28.37	0.02	5.22	0.10	0.95	0.79	0.78
			400	29.45	0.03	6.12	0.07	0.97	0.88	0.88
			1000	31.97	0.00	3.56	0.15	0.99	0.94	0.94
14	1.50	3.00	200	22.67	0.05	4.92	0.11	0.95	0.84	0.83
			400	24.62	0.04	5.74	0.08	0.97	0.91	0.91
			1000	31.13	0.01	3.47	0.15	0.99	0.94	0.94
15	1.50	4.00	200	18.22	0.10	4.36	0.13	0.95	0.88	0.87
			400	20.51	0.06	5.04	0.08	0.97	0.93	0.92
			1000	29.12	0.01	3.17	0.17	0.99	0.94	0.94
16	1.50	5.00	200	14.97	0.17	3.66	0.14	0.96	0.90	0.90
			400	16.98	0.12	4.05	0.12	0.98	0.94	0.94
			1000	27.08	0.01	2.78	0.19	0.99	0.95	0.95

All reported values are averages over $S = 1000$ replications.

[a] LR-test of rank r against the unrestricted model with $r = p$.

[b] $\hat{v}_{i,r=j}$ refers to the modulus of the i'th largest root of the companion matrix for the model with rank $r = j$. Thus, the first two columns are the two largest unrestricted roots of the companion matrix for the unrestricted model, while the final column is the largest unrestricted root in the reduced rank model with $r = 1$ (where the largest root is restricted to a unit root).

Table 8: Reduced Rank Estimations with $r = 1$: Cointegration Coefficients β

i	τ_b^i	c_μ^i	T	$\hat{\beta}_2$	$\hat{\beta}_2^*$	$se_{\hat{\beta}_2^*}$ [a]	$\tau_{\hat{\beta}_2^*}$ [b]	$\bar{b}_t$ [c]	$\hat{\beta}_2^* - \bar{b}_t$ [d]
1	0.25	2.00	200	-3.71	-2.88	0.74	-11.47	-2.00	-0.88
			400	-2.02	-2.02	0.19	-20.04	-2.00	-0.02
			1000	-2.06	-2.06	0.06	-40.19	-2.00	-0.06
2	0.25	3.00	200	-2.16	-2.16	0.63	-8.69	-2.00	-0.17
			400	-2.02	-2.02	0.34	-14.36	-2.00	-0.02
			1000	-2.06	-2.06	0.08	-30.37	-2.00	-0.06
3	0.25	4.00	200	-3.42	-1.90	1.12	-6.81	-2.00	0.10
			400	-2.19	-2.19	0.36	-10.11	-2.00	-0.19
			1000	-2.06	-2.06	0.11	-21.33	-2.00	-0.06
4	0.25	5.00	200	-0.46	-0.46	1.10	-5.92	-2.00	1.54
			400	-7.43	-2.52	0.79	-7.14	-2.00	-0.52
			1000	-2.03	-2.03	0.16	-14.59	-2.00	-0.03
5	0.50	2.00	200	-3.51	-4.91	1.26	-9.30	-2.00	-2.91
			400	-2.51	-2.51	0.27	-14.58	-2.00	-0.52
			1000	-2.12	-2.12	0.10	-23.13	-2.00	-0.12
6	0.50	3.00	200	-3.36	-1.68	0.92	-7.22	-2.00	0.32
			400	-2.63	-2.63	0.46	-11.26	-2.00	-0.64
			1000	-2.11	-2.11	0.12	-19.88	-2.00	-0.11
7	0.50	4.00	200	-4.54	-1.59	1.33	-5.87	-2.00	0.41
			400	-2.65	-2.65	0.79	-8.58	-2.00	-0.65
			1000	-2.09	-2.09	0.15	-16.14	-2.00	-0.09
8	0.50	5.00	200	-2.57	-0.67	1.24	-5.23	-2.00	1.33
			400	-2.62	-2.62	0.83	-6.56	-2.00	-0.63
			1000	-2.02	-2.02	0.18	-12.54	-2.00	-0.02
9	1.00	2.00	200	-2.77	-2.77	1.91	-6.37	-1.99	-0.78
			400	-6.62	-3.12	0.67	-8.77	-1.99	-1.13
			1000	-2.17	-2.17	0.19	-12.86	-2.00	-0.18
10	1.00	3.00	200	-1.86	-1.86	1.76	-5.02	-1.99	0.14
			400	-4.45	-3.00	1.06	-7.28	-1.99	-1.01
			1000	-2.13	-2.13	0.20	-12.20	-1.99	-0.13
11	1.00	4.00	200	-3.39	-3.39	1.77	-4.28	-1.99	-1.40
			400	1.42	-2.20	0.70	-6.02	-1.99	-0.21
			1000	-2.07	-2.07	0.21	-11.15	-2.00	-0.08
12	1.00	5.00	200	-1.39	-1.39	1.42	-4.00	-1.99	0.61
			400	-2.00	-2.00	0.97	-5.23	-1.99	-0.01
			1000	-1.94	-1.94	0.22	-9.84	-2.00	0.05
13	1.50	2.00	200	-5.77	-4.56	2.62	-4.94	-1.99	-2.57
			400	0.05	-2.88	1.20	-6.59	-1.98	-0.90
			1000	-2.17	-2.17	0.23	-10.18	-2.00	-0.18
14	1.50	3.00	200	-5.70	-3.68	2.56	-3.90	-1.99	-1.68
			400	-5.46	-3.30	0.90	-5.81	-1.98	-1.31
			1000	-2.14	-2.14	0.25	-9.89	-2.00	-0.15
15	1.50	4.00	200	-1.70	-1.70	2.07	-3.48	-2.00	0.30
			400	7.12	-1.44	1.14	-4.99	-1.99	0.54
			1000	-1.99	-1.99	0.24	-9.28	-2.00	0.01
16	1.50	5.00	200	-0.83	-0.83	1.38	-3.27	-2.00	1.16
			400	-0.93	-1.95	0.68	-4.51	-1.99	0.03
			1000	-1.89	-1.89	0.24	-8.68	-2.00	0.11

The column for $\hat{\beta}_2$ reports averages over $S = 1000$ replications. However, the columns for $\hat{\beta}_2^*$ report averages where a total of 18 out of the 48.000 estimates are excluded due to extreme estimates, where $\hat{\beta}_2 > 1.000$ or $\hat{\beta}_2 < -1.000$.

[a] Standard error of $\hat{\beta}_2$.

[b] T-value of $\hat{\beta}_2$.

[c] Sample average of the parameter b_t in the simulations.

[d] Difference between the estimated parameter $\hat{\beta}_2$ and the sample average of b_t.

Table 9: Reduced Rank Estimations with $r = 1$: Adjustment Coefficients α

i	τ_b^i	c_μ^i	T	$\hat{\alpha}_1$	$se_{\hat{\alpha}_1}$ [a]	$\tau_{\hat{\alpha}_1}$ [b]	$\hat{\alpha}_2$	$se_{\hat{\alpha}_2}$ [a]	$\tau_{\hat{\alpha}_2}$ [b]
1	0.25	2.00	200	0.16	0.04	4.34	0.17	0.03	4.69
			400	0.15	0.03	5.60	0.16	0.03	6.11
			1000	0.09	0.02	5.42	0.11	0.02	6.48
2	0.25	3.00	200	0.10	0.03	2.86	0.11	0.03	3.15
			400	0.10	0.02	4.31	0.11	0.02	4.71
			1000	0.07	0.02	4.48	0.08	0.01	5.40
3	0.25	4.00	200	0.05	0.04	1.53	0.06	0.03	1.82
			400	0.06	0.02	3.05	0.07	0.02	3.38
			1000	0.05	0.01	3.59	0.05	0.01	4.33
4	0.25	5.00	200	-0.00	0.04	0.22	0.02	0.04	0.54
			400	0.03	0.02	1.87	0.04	0.02	2.16
			1000	0.03	0.01	2.71	0.03	0.01	3.34
5	0.50	2.00	200	0.11	0.03	3.49	0.12	0.03	4.08
			400	0.09	0.02	4.15	0.11	0.02	5.15
			1000	0.03	0.01	2.34	0.05	0.01	4.51
6	0.50	3.00	200	0.07	0.03	2.23	0.08	0.03	2.74
			400	0.07	0.02	3.27	0.08	0.02	4.12
			1000	0.03	0.01	2.02	0.04	0.01	4.04
7	0.50	4.00	200	0.03	0.03	1.12	0.05	0.03	1.62
			400	0.04	0.02	2.24	0.05	0.02	2.98
			1000	0.02	0.01	1.68	0.03	0.01	3.49
8	0.50	5.00	200	-0.01	0.04	-0.03	0.02	0.03	0.49
			400	0.02	0.02	1.29	0.03	0.02	1.95
			1000	0.01	0.01	1.20	0.02	0.01	2.81
9	1.00	2.00	200	0.05	0.02	1.93	0.06	0.02	3.05
			400	0.03	0.02	1.68	0.05	0.01	3.73
			1000	-0.02	0.01	-1.26	0.02	0.01	2.75
10	1.00	3.00	200	0.02	0.02	1.05	0.05	0.02	2.05
			400	0.02	0.02	1.11	0.04	0.01	2.99
			1000	-0.02	0.01	-1.33	0.02	0.01	2.58
11	1.00	4.00	200	-0.00	0.03	0.24	0.02	0.02	1.22
			400	0.00	0.02	0.36	0.03	0.01	2.09
			1000	-0.02	0.01	-1.39	0.02	0.01	2.35
12	1.00	5.00	200	-0.03	0.03	-0.59	0.01	0.03	0.41
			400	-0.01	0.02	-0.27	0.02	0.01	1.37
			1000	-0.02	0.01	-1.58	0.01	0.01	1.92
13	1.50	2.00	200	0.02	0.02	0.96	0.04	0.01	2.53
			400	-0.01	0.02	-0.03	0.03	0.01	2.91
			1000	-0.04	0.01	-3.25	0.01	0.01	2.02
14	1.50	3.00	200	-0.00	0.02	0.15	0.03	0.02	1.61
			400	-0.02	0.02	-0.43	0.02	0.01	2.39
			1000	-0.04	0.01	-3.29	0.01	0.01	1.90
15	1.50	4.00	200	-0.02	0.03	-0.54	0.02	0.02	0.96
			400	-0.03	0.02	-1.05	0.02	0.01	1.58
			1000	-0.04	0.01	-3.29	0.01	0.01	1.73
16	1.50	5.00	200	-0.05	0.03	-1.23	0.01	0.02	0.27
			400	-0.03	0.02	-1.50	0.01	0.01	1.04
			1000	-0.04	0.01	-3.40	0.01	0.01	1.40

All reported values are averages over $S = 1000$ replications.
[a] Standard error of $\hat{\alpha}_1$ and $\hat{\alpha}_2$, respectively.
[b] T-value of $\hat{\alpha}_1$ and $\hat{\alpha}_2$, respectively

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