Preliminary Draft

A Keynes-IKE Model of Currency Risk: A CVAR Investigation*

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1 Introduction

A core puzzle in financial economics is the inability of standard risk-premium models to account for excess returns in currency and other asset markets.[1] These models' microfoundations rely on expected utility theory (EUT) and the rational expectations hypothesis (REH). EUT is based on an axiomatic approach that represents how an individual chooses among risky portfolios given her forecasts of their returns. In traditional portfolio-balance models, EUT implies that the one-period-ahead expected excess return - the risk premium - depends on the *ex ante* variance of returns. The rational expectations hypothesis (REH) is used to portray individuals' forecasts of the return and variance. REH assumes that these forecasts differ from *ex post* outcomes by white noise errors. This assumption is based on a premise that underpins much of modern macroeconomics: the process underpinning outcomes at every point in time can be represented adequately by a single, time-invariant, conditional probability distribution.

Research shows that both pillars of standard models lack empirical support. Many studies report experimental evidence that EUT's *a priori* assumptions about risk preferences are grossly inconsistent with the way individuals actually behave. There is also much evidence against REH's assumption of white-noise forecast errors.[2] Frydman and Goldberg (2007, 2008, 2013a) show that new ways of forecasting market outcomes, as well as new economic policies and other changes in the social context that cannot be fully foreseen, imply that any time-invariant statistical account of returns or forecast errors must eventually experience temporal instability at times and in ways that cannot be fully anticipated. Evidence of such temporal instability is overwhelming. Commenting in an interview with Institutional Investor on the temporal instability of correlations in asset-price data, Nobel laureate William Sharpe quipped that "[i]t's almost true that if you don't like an empirical result, if you can wait until somebody uses a different [time] period... you'll get a different answer" (Wallace, 1980, p. 24).

In this paper, we provide an empirical investigation of a portfolio-balance

[1] For a review article on the failure of standard theory in currency markets, see Lewis (1995) and Engel (1996). For other markets, see Campbell et al. (1997) and Seigel and Thaler (1997).

[2] For example, see Frankel and Froot (1989), Taylor (1989), Cavaglia et al. (1993, 1994), Madsen (1996), and Bacchetta, Mertens and van Wincoop (2009), who use survey data on exchange rate expectations and report strong rejections of REH.

risk premium model that jettisons both pillars of standard theory. The model, which is developed in Frydman and Goldberg (2007, 2013a), uses endogenous prospect theory to portray an individual's risk preferences and imperfect knowledge economics (IKE) to represent her forecasting behavior.[3]

Endogenous prospect theory implies that an individual's risk premium depends not on her expectation of the variance of returns, but on her point forecast of the potential loss that she might incur on her speculative position. In order to represent this forecast, the model builds on Keynes' (1936) insight that participants rely on a convention in assessing the riskiness of speculation: they relate the potential for capital loss to the gap between the asset price and their perceptions of its benchmark value.[4] Bulls, who hold long positions, tend to raise their forecasts of the potential loss as this gap grows, e.g. as the asset's price becomes more overvalued or less undervalued relative to the benchmark; whereas bears, who hold short positions, tend to respond in opposite fashion. The IKE representations of these forecasts leave open the size of the gap effect at any point in time. As such, the model does not imply a single, time-invariant, conditional probability distribution of returns. Nonetheless, the model implies a momentary equilibrium in which the market risk premium tends to co-move positively with the deviation between an asset's price and commonly-used measures of the benchmark value.[5]

We test this implication by applying a cointegrated VAR analysis to three major US dollar currency markets, the pound sterling, the Deutsche mark (DM), and yen. The CVAR enables us to nest the Keynes-IKE equilibrium relationship together with the equilibrium relationship that is implied by a

[3]Endogenous prospect theory extends Kahneman and Tversky's (1979) prospect theory to allow for heterogeneous expectations and imperfect knowledge in asset market models. IKE is an alternative approach to formal analysis enabling economists to recognize that the process underpinning market outcomes is to some extent open. Endogenous prospect theory and IKE are developed in Frydman and Goldberg (2007, 2008).

[4]Tobin (1958) also used this insight in modeling a precautionary motive for money demand. In this paper, he also develops the basic portfolio-balance approach, which relates financial risk to volatility. The profession picked up on the latter approach and completely ignored the former.

[5]Frydman and Goldberg (2007) show that the model also has implications for the frequency of sign reversals in the market risk premium, which, according to the model, should be lower during time periods in which the gap is relatively large. They find empirical support for this prediction. By contrast, standard portfolio-balance and consumption CAPM models have been unable to account for the sign reversals in the data. See Lewis (1996) and Mark and Wu (1998).

standard portfolio-balance model (Kouri, 1976, and Dornbusch, 1983). Our empirical analysis makes use of monthly survey data on exchange rate expectations from Money Market Services International (MMSI).[6]

These data enable us to measure *ex ante* returns directly, rather than relying on fitted values from regressions based on *ex post* returns, as is the case with other studies. Unfortunately, MMSI's monthly surveys did not collect participants' forecasts of the variance of returns. Consequently, to test the standard risk-premium model, we construct an REH measure of this *ex ante* variance using *ex post* data on daily realized currency returns one-month forward, as well as an adaptive measure using returns one-month prior.

To highlight our results, we find mean shifts in the cointegrating space for all three currency markets. These shifts occur at times and in ways that would be extremely difficult to anticipate fully. For example, the mean shift that is found for the DM market occurs in March 1991, which is proximate to German reunification. It is difficult to imagine how anyone in the early 1980's could have anticipated this development, let alone foreseen its exact timing and impact on the process underpinning returns in currency markets. Consequently, our results concerning structural change suggest a rejection of REH's presumption of a single probability distribution for currency returns. However, even if we were to entertain the possibility that one could have anticipated the timing and impact of the mean shifts, we reject the other key implication of the standard model: we find little to no evidence in the three currency markets of an equilibrium relationship between the market risk premium and our REH measure of the *ex ante* variance of returns.

By contrast, we find strong support for the main prediction of the Keynes-IKE model. Our cointegration results show a positive equilibrium relationship between the market risk premium and the gap between the exchange rate and its purchasing power parity (PPP) value in all three currency markets. Surprisingly, we do find that forward- and backward-looking measures of the ex ante variance of returns do play a role, but primarily in the short-run component of the model and typically occurring *through* the gap equation.

[6] MMSI and other survey data on exchange rate expectations have been used extensively in the literature, largely to test REH's implication of white noise forecast errors. For references, see footnote 2. Frydman and Goldberg (2007) is the only other study that we know of in the exchange rate literature that uses the survey data to test directly the implications of risk premium models. See Fuhrer (2013), who uses survey data to test the implications of New Keynesian models of output, interest rates, and inflation.

This suggests that the inability of the previous literature to detect an effect of volatility on the premium was, in part, an omitted variable bias of failing to control for this gap effect. This result points to a need to extend the Keynes-IKE model, perhaps by using both EUT and endogenous prospect theory to represent risk preferences.[7]

2 The Keynes-IKE Risk-Premium Model

Like traditional portfolio-balance models of currency returns, the Keynes-IKE model assumes that market participants choose at each point in time the proportion of domestic and foreign bonds they should hold so as to maximize next period's utility. This decision depends on participants' preferences and their forecasts of the return on foreign exchange.

2.1 A New Specification of the Risk Premium

The Keynes-IKE model uses endogenous prospect theory to represent a participant's preferences and decision rule. One of the key assumptions of endogenous prospect theory is that an individual's degree of loss aversion increases with the size of their open positions in the market.[8] This assumption of "endogenous loss aversion" implies that market participants hold finite speculative positions in foreign exchange only if they expect a positive return – a risk premium – to compensate them for their extra sensitivity to the potential losses.[9]

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[7] Barberis et al. (2001) provides an example of such a mixing of approaches in the context of an REH model of returns in stock markets. However, the study does not attempt to account for the actual time path of the ex ante excess return, as we do here, instead focusing on the so-called equity-premium puzzle. In specifying preferences, Barberis et al. (2001) incorporates only one of the key assumptions of prospect theory — loss aversion — so that prospect utility can be specified in terms of expected values. Endogenous prospect theory allows us to maintain all of Kahneman and Tversky's findings, including diminishing sensitivity, in modeling the speculative decision in asset markets.

[8] Kahneman and Tversky's (1979) prospect theory assumes that individuals are "loss averse": the disutility that they would experience from a loss exceeds the utility from gains of the same magnitude.

[9] Behavioral-finance researchers refer to an individual's decision to hold a finite speculative position as "limits to arbitrage," which they consider to be one of the pillars of their approach (Barberis and Thaler, 2001). Endogenous prospect theory provides a way to model limits to speculation without abandoning any of the experimental findings of

Endogenous prospect theory leads to a new specification for an individual's risk premium, which depends on her forecast of the potential losses. For the group of bulls, this premium, which Frydman and Goldberg (2007) call an "uncertainty premium", can be written as follows:

$$\widehat{up}^{i}_{t|t+1} = (1-\lambda_1)\,\widehat{l}^{i}_{t|t+1} > 0..i = \text{L,S} \tag{1}$$

where $\widehat{l}^{i}_{t|t+1}$ represents an aggregate of bulls' or bears' time-t point forecasts of the potential loss from holding a unit-sized open position for one period, and superscripts L and S denote long and short position, respectively.[10]

An individual's time-t forecast of the potential unit loss at $t+1$ is portrayed by the expected value of the "loss part" of a probability distribution for the one-period return on an open position.[11] For bulls, the "expected unit loss" is,

$$\widehat{l}^{\text{L}}_{t|t+1} = E^{\text{L}}_{t}[r_{t+1}|r_{t+1} < 0, Z^{\text{L}}_{t}] < 0 \tag{2}$$

while for a bear we have,

$$\widehat{l}^{\text{S}}_{t|t+1} = -E^{\text{S}}_{t}[r_{t+1}|r_{t+1} > 0, Z^{\text{S}}_{t}] < 0 \tag{3}$$

The point forecasts in (2) and (3) are conditional on individuals' forecasting strategies and information sets, Z^{i}_{t}. The one-period excess return on a long position in foreign exchange,

$$r_{t+1} = s_{t+1} - s_t + i^*_t - i_t \tag{4}$$

is expressed using a log approximation, where s_t denotes the log spot exchange rate and i_t and i^*_t are the nominal returns on domestic and foreign bonds, respectively. As such, the one-period return on a short position is given by $-r_{t+1}$. We note that losses for bulls (bears) involve negative (positive) realizations of r_{t+1}. Hence, the negative sign on $E^{\text{S}}_{t}[\cdot]$ in expression (3).

Risk in the model depends only on the loss part of the probability distribution that is used to represent the group of bulls' or bears' forecasting

Kahneman and Tversky (1979) and others.

[10] The uncertainty premium in (1) is the minimum expected return that an individual requires in order to hold an open position in the market. The term, uncertainty premium, highlights Knight's (1921) distinction between uncertainty and risk, which recognizes that the risk in markets stems from the inherent imperfection of knowledge.

[11] As news arrives, an individual may revise her strategy for forecasting potential losses. The model represents the new forecasting strategy with a different probability distribution.

strategy, rather than on both the loss and gain parts as is the case with standard volatility measures. A higher $\widehat{l}^{i}_{t|t+1}$ implies that individuals attach a greater risk of capital loss to speculating. Equation (1) shows that because individuals are endogenously loss averse ($\lambda_1 > 1$), a higher $\widehat{l}^{i}_{t|t+1}$, meaning a greater negative value, leads them to raise their uncertainty premium.

Endogenous prospect theory and portfolio balance lead to a new *momentary* equilibrium condition for the currency market. It is obtained by aggregating individuals' demands and supplies for foreign exchange using wealth shares and assuming that the exchange rate adjusts instantaneously to balance the total of buying and selling in the market at every point in time:

$$\widehat{r}_{t|t+1} = \widehat{up}_{t|t+1} + \lambda_2 IFP_t \tag{5}$$

where $\widehat{r}_{t|t+1} = \hat{s}_{t|t+1} - s_t + i^*_t - i_t$, $\hat{s}_{t|t+1}$ represents the aggregate of participants' conditional point forecasts of s_{t+1}, IFP_t is the international financial position of the domestic country relative to the foreign country, $\lambda_2 > 0$ is another preference parameter, and $\widehat{up}_{t|t+1}$ is the aggregate uncertainty premium,

$$\widehat{up}_{t|t+1} = \widehat{up}^{\mathrm{L}}_{t|t+1} - \widehat{up}^{\mathrm{S}}_{t|t+1} = \frac{1}{2}\left(1 - \lambda_1\right)\left(\widehat{l}^{\mathrm{L}}_{t|t+1} - \widehat{l}^{\mathrm{S}}_{t|t+1}\right) \tag{6}$$

which depends on the uncertainty premium of bulls minus the uncertainty premium of bears.[12]

According to equation (5), momentary equilibrium is obtained when the expected return, $\widehat{r}_{t|t+1}$, offsets the uncertainty premium, $\widehat{up}_{t|t+1}$, sufficiently so that market participants in the aggregate willingly hold the available supplies of foreign and domestic bonds. The implied market premium – $\widehat{pr}_{t|t+1} = \widehat{up}_{t|t+1} + \lambda_2 IFP_t$ – depends on both the aggregate uncertainty premium and asset supplies.

2.2 Connecting Currency Risk to Perceptions of the Gap

In order to represent bulls' and bears' forecasts of the potential unit loss, Frydman and Goldberg (2007) appeal to an insight from Keynes (1936), that what matters for assessing risk in financial markets is the divergence between

[12] In deriving equation (6), Frydman and Goldberg, like Delong et al. (1990), assume that the wealth share of bulls is constant and equal to that of bears, thereby implying that $\widehat{up}^{\mathrm{L}}_{t|t+1}$ and $\widehat{up}^{\mathrm{S}}_{t|t+1}$ enter the market premium with equal weights.

an asset's price and its perceived historical benchmark value. Although asset prices have a tendency to move persistently away from benchmark values for long stretches of time, they eventually undergo, at unpredictable moments, sustained movements back toward these values. Keynes recognized that market participants are aware of this regularity and use it in their attempt to assess the riskiness of their open positions. As he put it in discussing the bond market,

> [u]nless reasons are believed to exist why future experience will be very different from past experience, a ...rate of interest [much lower than the benchmark rate], leaves more to fear than to hope, and offers, at the same time, a running yield which is only sufficient to offset a very small measure of fear [of capital loss] (Keynes, 1936, p.202).

The model formalizes Keynes's insight with the following specification for bulls' and bears' forecasting strategies for the potential unit loss from speculating:

$$\widehat{l}^{i}_{t|t+1} = \mu_t + \delta^{i}_{t}\widehat{gap}_t + \varepsilon^{i}_{t} \quad i = \text{L,S} \tag{7}$$

where $\mu_t < 0$ is a mean value, $\delta^{\text{L}}_{t} < 0$ for bulls and $\delta^{\text{S}}_{t} > 0$ for bears, $\widehat{gap}_t = s_t - \hat{s}^{\text{BM}}_{t}$, $\hat{s}^{\text{BM}}_{t}$ is the perceived benchmark value, and ε_t is an error term that represents the influence of factors other than the gap on $\widehat{l}^{i}_{t|t+1}$, which are assumed not to have a systematic effect. The t subscripts on the parameters in (7) recognize that participants may revise how they interpret $\widehat{gap}_t$ in forecasting potential losses, at least intermittently, over time. We recall that $\widehat{l}^{i}_{t|t+1}$ is negative for both bulls and bears, so a negative δ^{L}_{t} and positive δ^{S}_{t} reflect Keynes's insight that a rising $\widehat{gap}_t$ leads bulls to increase and bears to decrease their forecasts of the *size* of the potential unit loss from speculating. We assume that the size of μ_t is sufficiently large to ensure that $\widehat{l}^{i}_{t|t+1} < 0$ regardless of how $\widehat{gap}_t$ varies. We note that, in general, market participants have diverse notions of the benchmark value. However, whatever their notion, their estimates of the benchmark value vary much less than the exchange rate itself. Consequently, movements in a participants' estimate of $\widehat{gap}_t$ will be dominated by movements in s_t no matter how they estimate $\hat{s}^{\text{BM}}_{t}$. And since the time series implications of the model depend on how $\widehat{gap}_t$ varies over time, we abstract from differences in estimates of $\hat{s}^{\text{BM}}_{t}$.

With the specification in (7), we can write the aggregate uncertainty premium as:

$$\widehat{up}_{t|t+1} = \rho_t + \sigma_t \widehat{gap}_t + \varepsilon_t \tag{8}$$

where $\rho_t = \frac{1}{2}(1-\lambda_1)\mu_t$, $\sigma_t = \frac{1}{2}(1-\lambda_1)(\delta_t^{\mathrm{L}} - \delta_t^{\mathrm{S}}) > 0$, and ε_t depends on the errors in (7). In order to derive time series implications from the model, we need to represent how bulls and bears might revise their strategies for forecasting the potential unit loss, that is, we need restrictions on how the parameters μ_t and δ_t^i change over time.

2.3 IKE Constraints on Change and Their Time Series Implications

The IKE constraints that the model imposes on this change recognize that no one, including economists, can fully anticipate when and how market participants might decide to revise how they interpret the gap in forecasting potential losses. Indeed, Frydman and Goldberg (2007, 2011) present evidence that the importance individuals attach to the gap when it is historically large is greater than when it is historically small. Market participants themselves, let alone economists, cannot fully foresee the thresholds above or below which they might consider the magnitude of the gap to be large or small or how the crossing of these thresholds might impact their forecasts of the potential losses.

In modeling such change, the model again appeals to Keynes's (1936) account of asset markets. In using their "knowledge of the facts" to form forecasts, participants

> "fall back on what is, in truth, a convention... [which] lies in assuming that the existing state of affairs will continue indefinitely, except in so far as we have specific reasons to expect a change." (Keynes, 1936, p. 152)[13]

This insight suggests that market participants tend to stick with a forecasting strategy for stretches of time. Indeed, it is often unclear whether one should alter her strategy. A quarter or two of poor forecasting performance may be the result of random events rather than an indication of a failing strategy. So, unless an individual has "specific reasons to expect a change" in the market,

[13] By "existing state of affairs," Keynes means "knowledge of the facts."

she may leave her current strategy unaltered – even if its performance begins to flag over several periods. Moreover, even armed with "specific reasons to expect a change," it is entirely unclear what new forecasting strategy, if any, she should adopt.

The Keynes-IKE model represents how participants alter their thinking about how potential losses are related to the gap with a contingent regularity that Frydman and Goldberg (2007) call "guardedly moderate revisions": there are stretches of time during which participants either maintain their strategies or revise them gradually. It is clear from equation (8) that any stretch of time in which market participants in the aggregate kept their forecasting strategies unchanged would involve a stable positive relationship between the aggregate uncertainty premium and the aggregate gap. Moreover, if a stretch of time also involved some points at which revisions of strategies were sufficiently moderate, the model would continue to imply a positive co-movement between $\widehat{up}_{t|t+1}$ and $\widehat{gap}_t$, although such change would lead to shifts in the parameters of the relationship in (8).[14]

But, although market participants have a tendency to maintain their strategies or revise them gradually, this qualitative regularity is contingent: it manifests itself at times and in ways that no one can fully foresee. There are occasions when price movements and news about economic and political developments lead participants to revise their forecasting strategies in non-moderate ways. Such revisions can have a dramatic impact on the relationship between $\widehat{up}_{t|t+1}$ and $\widehat{gap}_t$ and may imply no or even negative comovements in these variables.

The Keynes-IKE constraints on change are thus qualitative and contingent. Nonetheless, the model predicts that if participants' tendency to stick with their strategies or revise them gradually is pronounced enough, $\widehat{up}_{t|t+1}$ and $\widehat{gap}_t$ will tend to co-move positively over time.

[14]The condition that ensures a gap effect is $\left|\boldsymbol{\delta}^i_{t-1}\Delta\widehat{gap}_t\right| > \left|\Delta\mu^i_t + \Delta\delta^i_t\widehat{gap}_t\right|$, where $|\cdot|$ denotes an absolute value. For more discussion on such a guardedly moderate constraint, see Frydman and Goldberg (2007, 2013b).

2.4 An IKE Econometric Specification

This prediction can be expressed in the context of a time-varying error-correction model. Equations (5) and (8) imply the following temporary error-correction specification for each stretch of time in which participants largely maintain their strategies:

$$\Delta\widehat{r}_{t|t+1} = \alpha\left[\widehat{r}_{t-1|t} - \sigma^{j}\widehat{gap}_{t-1} - \lambda_2 IFP_{t-1} - \rho^{j}\right] + \sigma^{j}\Delta\widehat{gap}_{t} + \lambda_2\Delta IFP_t + \varepsilon_t \tag{9}$$

where $\alpha = -1$ and $j = 1, 2..$ denotes distinct stretches of time in the data for which the models' parameters are relatively stable. During each of these stretches, the model implies a temporary cointegrating vector that embodies a gap effect ($\sigma^j > 0$). The Keynes-IKE model implies that short-run movements of $\widehat{r}_{t-1|t}$ in each distinct linear piece of the data involve a quick return back to the temporary cointegrating vector: with $\alpha = -1$, the system tends to move back to momentary equilibrium the very next period barring further shocks.

In order to test for points of structural change in the model, we rely on the recursive procedure of Hansen and Johansen (1999) referred to as the eigenvalue fluctuation test. We simplify the analysis by representing change in the process underpinning excess returns with mean shifts in the cointegrating space (that is, in ρ^j), while assuming a stable gap effect over the entire sample periods.[15]

2.5 A Graphical Inspection of the Data

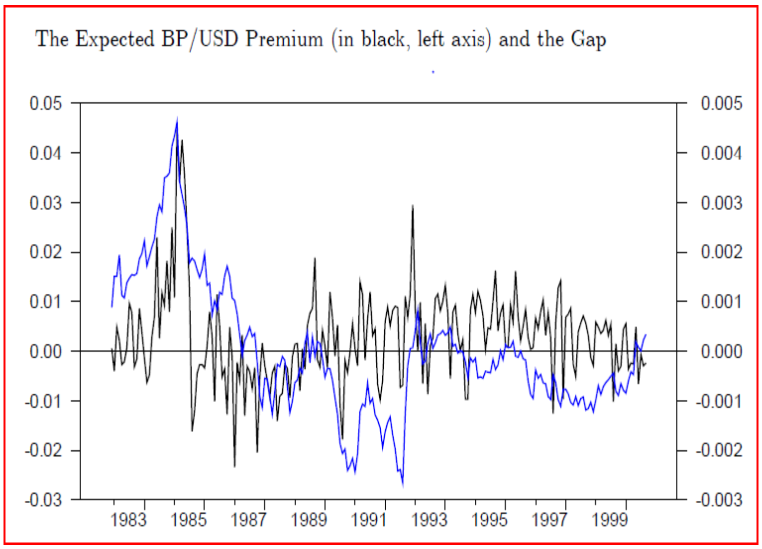
Figure 1 plots the *ex ante* excess return on holding U.S. dollar long positions in the British pound-dollar (BP/$) market and the gap between the exchange rate and its purchasing power parity (PPP) level, which is a commonly used measure of the benchmark value.[16] The tendency for the market premium to co-move positively with the gap from PPP is striking.

[15] Hendry (2000) demonstrates that mean shifts in the cointegrating space are generally easier to detect than instability in the dynamic components of the model. In Frydman et al. (2013), we also allow for shifts in σ^j.

[16] Our PPP benchmark is calculated using the *Big Mac* PPP exchange rate reported in the April 6, 1990 issue of *The Economist* magazine (which was BP1.96/$1) and CPI-inflation-rate differentials from the IMF's *International Financial Statistics*. PPP exchange rates have long traditions as benchmark values in currency markets. See Rogoff (1996), Sarno and Taylor (2002), and Taylor and Taylor (2004).

However, the figure also shows that $\widehat{r}_{t|t+1}$ is more volatile than $\widehat{gap}_t$, suggesting that there may be other risk factors of relevance aside from the gap, perhaps including the volatility measure.

Figure 1: The BP/USD Premium (in black) and the Gap (in blue)



3 An REH Risk Premium Model

The International Capital Asset Pricing Model (CAPM) is well-known and so we omit the full derivation and present only the equilibrium condition implied by EUT and portfolio balance, which we will refer to as Risk-adjusted Uncovered Interest Parity (RAUIP):

$$\widehat{r}_{t|t+1} = \phi \hat{\nu}_{t|t+1} IFP_t \tag{10}$$

where $\hat{\nu}_{t|t+1}$ denotes the *ex ante* variance of changes in the spot exchange rate, ϕ is the coefficient of relative risk aversion, and IFP_t is defined as before. The intuition of the model is that, following EUT, portfolios possessing a greater variance require a higher expected return to compensate for their inherent additional risk. Under the assumption of deterministic inflation rates, typically justified by the far greater variance of the exchange rate compared to relative prices, the minimum variance portfolio would be comprised of solely domestic bonds (as they possess no vulnerability to exchange rate risk). Investors then require a premium to hold foreign assets, which is an increasing function of the variance of changes in the exchange rate. The sign of the market premium is determined by the net foreign asset holdings, or the international financial position. If one country is a net debtor, this requires foreign investors to hold the (from their perspective) riskier asset, necessitating a premium to induce them to do so. If IFP_t is positive, this means the domestic country is a net creditor and the expected foreign excess return $\widehat{r}_{t|t+1}$ will likewise be positive.

Empirical work testing the International CAPM has typically involved estimating versions of the above equation for risk-adjusted uncovered interest parity (RAUIP) with *ex post* data on excess returns, assuming REH, and evaluating it based on two criteria. The first is to test the implied restriction of mean variance optimization against the more general model where the coefficient on the IFP_t term is an unrestricted, time-varying parameter. A second important criterion is to examine the estimates of ϕ. Theory implies that it should be positive and statistically significant, but it also needs to fall within what is generally regarded as a reasonably low range. Mehra and Prescott (1985) for example did not consider estimates higher than 10, while others argue in favor of the "Samuelson presumption" that a reasonable estimate would be around two (Krugman 1981). This corresponds to an individual being indifferent between a 4% loss, and a gamble equally likely to produce a gain or loss of 20%.

Most of these studies proxy the IFP_t term via outstanding government bonds. The earliest studies assumed that the variance was constant, and then tested whether government bond supplies could explain *ex post* excess returns. Lewis (1988a) finds very little explanatory power in the model, and in fact for two of the four countries obtains the wrong sign (an increase in the asset supply is associated with a decrease in the return).

In order to introduce more variability in the relationship, studies have allowed the variance to vary over time. To this end they have related this variance to fundamental variables or modeled it using options prices or by assuming an auto-regressive conditional heteroskedastic (ARCH) process. Engel (1996) summarizes that the performance of these models has largely been disappointing, obtaining primarily insignificant or negative estimates.

A more recent popular alternative for estimating a time-varying variance is to use what is generally referred to as a "realized volatility measure," constructing the measure from the intra-period observations based on ex post data.[17] Recent research has shown this to be a highly efficient and "model free" procedure for estimating the variance (it is non-parametric and can be treated as observable rather than latent), which often outperforms the earlier ARCH techniques which omit intra-period information. In this work, two realized alternatives are tested, an REH measure, using the variance actually observed one-period ahead, and a backward-looking measure based on the previous month's variance.

In order to arrive at the estimated functional form, we take the first-order Taylor approximation of $\phi\hat{\nu}_{t|t+1}IFP_t$. The expression reduces to:

$$\phi\hat{\nu}_{t|t+1}IFP_t = c_1 + c_2 IFP_t + c_3\hat{\nu}_{t|t+1} \tag{11}$$

c_1 is a constant representing the value of the function at the point of linearization $(\hat{\nu}_0, IFP_0)$; the second and third terms respectively are the derivatives with respect to $\hat{\nu}_{t|t+1}$ and IFP_t; and $c_2 = \phi(\hat{\nu}_{t|t+1} - \hat{\nu}_0)$ and $c_3 = \phi(IFP_t - IFP_0)$.

Again with a bit of algebra, the international CAPM model can be rewritten in terms of an error-correction formulation:

$$\Delta\widehat{r}_{t|t+1} = \alpha\left[\widehat{r}_{t-1|t} - c_3\hat{\nu}_{t-1|t} - c_2 IFP_{t-1} - c1\right] + c_3\Delta\hat{\nu}_{t|t+1} + c_2\Delta IFP_t + \varepsilon_t \tag{12}$$

The theoretical model again predicts an adjustment coefficient $\alpha = -1$.

4 The Cointegrated VAR Model

The cointegrated VAR (CVAR) model enables us to nest the international CAPM and IKE gap model in one empirical specification, providing the first direct comparison and joint estimation. Testing between the models is conducted with differing over-identifying restrictions on the CVAR discussed in the following two sections. Lastly, we can estimate a hybrid model allowing for the effects of both volatility and the gap simultaneously.

$$\Delta\widehat{r}_{t|t+1} = \alpha\left[\widehat{r}_{t-1|t} - c_3\hat{\nu}_{t-1|t} - \sigma^j\widehat{gap}_{t-1} - \mu_t\right] + c_3\Delta\hat{\nu}_{t|t+1} + \sigma^j\Delta\widehat{gap}_t + \varepsilon_t \tag{13}$$

[17] See French, Schwert and Stambaugh (1987) for monthly measures, and Andersen, Bollerslev, Diebold and Labys (2001) and Berndorff-Nielsen and Shephard (2001a, 2002) for daily measures.

where μ_t captures the deterministic components of both models, $c1$ and ρ^j, as well as IFP_t.

Monthly data on the bi-lateral international financial position between countries is not available. However, the very slow trending nature of movements in the international financial position of the U.S. and other advanced countries vis-a-vis the rest of the world suggests that bi-lateral positions are also slowly trending. To account for the influence of this variable, therefore, we allow for piece-wise deterministic trends in the model.

The CVAR model (Johansen 1989, 1991) extends the error-correction model of Engel and Granger (1987) to allow for a systems approach with mulitple, simultaneous cointegrating relations. The data is ordered in terms of the levels of persistence.[18] The ECM for a VAR(2) model, which two lags, can be represented generically as:

$$\Delta x_t = \Gamma \Delta x_{t-1} + \Pi x_{t-1} + \mu_t + \varepsilon_t \tag{14}$$

where the vector $x_t' = [\hat{s}^e_{t+1|t} - s_t, i_t, i_t^*, gap_t, \Delta p_t, \Delta p_t^* | \hat{\nu}_{t|t+1}]$ denotes respectively the expected change in the spot exchange rate (as measured by survey data), the domestic and foreign interest rates, the gap or real exchange rate, and the domestic and foreign inflation rates. The model is also conditioned on the volatility measure when testing the traditional model, and the hybrid model allowing for an effect of gap_t and $\hat{\nu}_{t|t+1}$. The Π matrix is just a reformulation of the covariances in the data, while Γ represents the coefficients of the short-run dynamics. μ_t represents the deterministic components of the model (constant, mean shifts, or break trends etc.), and ε_t is an i.i.d. error term.

If the variables in the information set are integrated of order 1 (I(1)), the unit roots or common stochastic trends imply that the matrix Π is not full rank. When the matrix is reduced rank, it can be decomposed into an α vector and a β' vector. The β' vector describes the linear combinations of the variables which become stationary. The β' vectors are interpreted as representing an equilibrium between the variables. The α vector meanwhile describes the error-correction mechanism indicating which variables are endogenous and adjust back to equilibrium following shocks.

The cointegrated VAR is designed to allow the data to "speak freely" in terms of the rank (number of relationships in the information set), and the

[18] See Johansen (1996) and Juselius (2006) for book-length treatments of the CVAR model.

pulling and pushing forces of the system (which variables are error-correcting and which are weakly exogenous driving the equilibrium), rather than constraining the data from the outset with untested assumptions concerning the rank and causation.[19]

Identification in the CVAR is achieved by imposing restrictions on the coefficients of the cointegrating relationships. This is accomplished by focusing on the expected return, imposing two symmetry restrictions that imply that the coefficients on the expected change in the exchange rate and the two interest rates are equal, though with opposite signs for the domestic interest rate, and are normalized to one. These restrictions lead to the expected excess return on foreign exchange $\hat{s}^e_{t|t+1} - s_t + i^*_t - i_t$ as a variable in the model, and by restricting the inflation rates to zero in the premium relationships, over-identification is achieved and the standard errors and stationarity of the relationships can be estimated.

The volatility measure possesses a large positive skewness, due to not only the preponderance of large positive shocks, but also the dearth of large negative shocks. In turn, to achieve a statistically well-specified model, meeting the requisite properties for valid statistical inference, the relationship is conditioned on this volatility measure rather than incorporating it with long-run feedback.

Tests for parameter stability indicate the need for a break in level, or mean shift, in the cointegrating space for the DM and BP sample. This allows for a change in the constant term after the German reunification in 1991:03 and to the inflation series for the UK in 1991:04. Dummy variables are also included for the months following the abandonment of the European Monetary System though the results are robust to their exclusion. A broken linear trend in the Yen sample in 1993:01 is also found, which could be connected to the worsening of the US bilateral current account vis-a-vis Japan. Failing to allow for such change yields a model which is not as statistically well-specified and renders inference dubious. MacDonald and Juselius (2004) also report a mean shift in the cointegrating space for the DM sample at the same time in their study, which uses a nearly identical information set, though without the survey forecast variable.

[19] This term "speak freely" comes from Hoover, Johansen, and Juselius (2006). See also Hendry and Mizon (1993) for more on the general-to-specific methodology of the CVAR.

5 Results

5.1 Tests of Risk-Adjusted and Uncertainty-Adjusted UIP

Table 1 reports the results for the cointegrating relationships. One can think of the premium as the left-hand side variable in all models, though the precise nature of the endogeneity will be examined in the following sub-section. Three models are then tested for each of the three exchange rate samples. The first is the ability of volatility to account for the premium (RAUIP), the second tests the gap's ability (UAUIP), and the last is a model including both effects (referred to as the hybrid model). The CVAR model allows one to nest all of these models in one empirical specification, and to test across them with the use of alternative restrictions. In all cases the premium restriction is imposed for identification, and the inflation rates are restricted to zero to achieve over-identification.

In the RAUIP models, the month-ahead volatility measure is given a free parameter, while the gap is restricted to zero, and vice versa for the gap model. Both are given a free parameter in the hybrid model allowing for both effects. The table presents for each of the three exchange rate samples coefficient estimates, with the t-value in parentheses below them, and the p-value for the likelihood ratio test of the restrictions and stationarity for each model. The results here are using the 10-year government bond rates, denoted by b.

Table 1: The Cointegrating Relations
Left hand side variable - the premium $\hat{s}^e_{t+1|t} - s_t + b^*_t - b_t$

$Sample$	β $vector$	$\hat{v}_{lead}$	gap_t	$const.$	$mean\ shift$	$p-value$
DM	$RAUIP$	0.981	0	0.001	-0.006	0.009
		(2.682)	–	(0.695)	(-2.211)	
	$UAUIP$	0	2.660	0.001	-0.012	0.514
		–	(4.563)	(0.480)	(-4.866)	
	$Hybrid$	1.368	2.667	0.001	-0.011	0.338
		(2.692)	(3.560)	(0.499)	(-4.470)	
BP	$RAUIP$	0.349	0	0.002	-0.003	0.012
		(1.719)	–	(0.810)	(-1.485)	
	$UAUIP$	0	2.971	0.001	-0.006	0.633
		–	(3.782)	(.587)	(-2.828)	
	$Hybrid$	3.193	2.708	0.004	-0.007	0.386
		(2.052)	(3.607)	(1.813)	(-3.346)	
JY	$RAUIP$	0.162	0	0.000	-0.000	0.224
		(2.379)	–	(0.520)	(-0.362)	
	$UAUIP$	0	1.888	0.001	-0.000	0.478
		–	(3.405)	(.475)	(-2.621)	
	$Hybrid$	0.052	2.013	0.001	-0.013	0.289
		(2.373)	(2.739)	(.819)	(-6.977)	

A few general results emerge, which are robust to alternative modeling specifications.[20] The first is that in all cases the UAUIP relationship, which includes the gap effect, is stationary, and more stationary than the RAUIP relationship which includes volatility. This implies that UAUIP provides a superior account of the equilibrium risk premium. In all cases, the gap variable is positive and significant, corroborating the hypothesis of the Keynes-IKE gap model. This positive coefficient tends to reject any explanations which imply a counter-cyclical risk premium. Interestingly, the coefficient on the gap variable as measured is very similar to the degree of loss aversion estimated by Kahneman and Tversky (1979), in a range around 2.25 (1.89-2.97 across the three samples). This is also true for the alternative modeling

[20] Further output and robustness tests for exhaustive combinations using short and long term interest rates, lagged or leading volatility, and separate or nested testing can be found in the appendix of a longer version of this paper available at http://wsbe.unh.edu/josh-r-stillwagon.

specifications included in the extended version of this paper.

When using the month-ahead realized volatility measure, the variable is of the correct sign, and significant, whereas there has been no such connection in the majority of the literature. While it is not quite significant in the case of the BP sample at 5%, it does become so once simultaneously incorporating the gap variable. The RAUIP relationships however are strongly rejected as stationary in the case of the DM and BP samples, suggesting that this does not constitute a sufficient equilibrium relationship to describe the data adequately. The relationship does become stationary however after adding the gap term in the hybrid model, but incorporation of the volatility series actually lowers the p-value of the relationship compared to that solely including the gap. The p-value can be interpreted similarly to an adjusted R-squared, and this reduction in the p-value from incorporation of volatility into the gap model implies that its value in providing additional information about the long-run cointegrating relationship with the premium does not outweigh the cost of estimating additional parameters in the model. It is also worth noting that the mean shift is statistically significant in all of the stationary relations with the one exception of the RAUIP relation for the yen sample.

5.2 Error-Correction

Table 2 provides the results for the α vector, or pulling forces which adjust back to any disequilibrium. Error-correction is implied by a significant coefficient for the change in a variable (based on the t-values in parentheses below), and with the opposite sign to that in the β vector. The results exclude those for the interest rates, which were almost all insignificant, and in the one exception was of very small magnitude ($-.007$ for the BP volatility model). The tests of weak exogeneity support the finding in the alpha vectors that the interest rates are not adjusting to disequilibrium. The volatility measure is also automatically excluded since the models are conditioning on it due to the skewness of the measure, rather than incorporating it with long-run feedback.

Table 2: Error-Correction

$Sample$	α $vector$	$\Delta(\hat{s}^e_{t+1\|t} - s_t)$	$\Delta\Delta p_t$	$\Delta\Delta p^*_t$	Δgap_t
DM	$RAUIP$	-0.605	0.291	-0.766	0.007
		(-0.819)	(2.572)	(-8.376)	(0.334)
	$UAUIP$	-0.696	-0.118	0.051	0.005
		(-5.894)	(-6.392)	(3.441)	(1.670)
	$Hybrid$	-0.621	0.209	-0.543	0.008
		(-1.189)	(-2.560)	(-8.248)	(0.538)
BP	$RAUIP$	-0.408	-0.013	-0.032	-0.004
		(-5.499)	(-0.542)	(-2.016)	(-1.471)
	$UAUIP$	-0.501	-0.011	0.001	-0.003
		(-6.760)	(-0.468)	(0.056)	(-0.845)
	$Hybrid$	-0.529	-0.022	-0.045	-0.003
		(-6.296)	(-0.818)	(-2.463)	(-0.908)
JY	$RAUIP$	-0.750	-0.039	0.262	0.004
		(-3.398)	(-0.778)	(10.057)	(0.803)
	$UAUIP$	-0.416	-0.275	0.072	0.004
		(-3.477)	(-6.521)	(3.483)	(0.967)
	$Hybrid$	-0.367	0.864	-0.440	-0.003
		(-0.661)	(-9.001)	(-2.248)	(-0.121)

The results on the error-correction are fairly consistent across the samples. The largest magnitude adjustment is often through the expected change in the exchange rate $\hat{s}^e_{t+1|t} - s_t$. The estimate varies between -0.35 and -0.75, implying an equilibrium correction within one (monthly) period of between roughly one-third and three-fourths, barring further shocks. A coefficient of -0.5 would imply that the half-life of deviations from UAUIP is one month. This demonstrates that the half-life of deviations from UAUIP are estimated in the range of one to two months or less.

5.3 Short-Run Dynamics of Volatility

While volatility appeared to have little relevance in understanding the movements of the long-run equilibrium premium, as defined by the cointegrating relations, it does appear to have some significant impacts on the short-run dynamics. This implies that it is the innovation to volatility which affects the changes in the other variables, as opposed to the level of volatility mattering for the level of the other variables. Table 3 shows the results of the short-run dynamics from the joint model including the gap and volatility. The number of lags included depend on the VAR structure of the model. Conditioning on the volatility measure in the VAR(3) model for example, as was used in the JY sample, includes a contemporaneous effect at time t, and two lagged effects at time $t-1$ and $t-2$, whereas the $t-2$ effect is excluded from a VAR(2) model as is the case for the DM and BP samples. All the models then include an effect of the month-ahead volatility and the month prior.The results for the bond rates are excluded, as the results were almost wholly insignificant, with one exception of a contemporaneous effect on the Japanese bond rate, though the magnitude was rather low (-0.009).

Table 3: Volatility Effects on the Short-Run Dynamics

	$\Delta(\hat{s}^e_{t+1\|t} - s_t)$	$\Delta\Delta p_t$	$\Delta\Delta p^*_t$	Δgap_t
$DM\Delta\hat{v}$	2.093	-0.456	0.362	0.035
	(1.552)	(-2.167)	(2.128)	(0.946)
$DM\Delta\hat{v}_{-1}$	0.236	-0.016	-0.389	0.038
	(0.180)	(-0.076)	(-2.350)	(1.039)
$BP\Delta\hat{v}$	0.900	0.108	-0.099	0.063
	(1.161)	(0.424)	(-0.589)	(1.953)
$BP\Delta\hat{v}_{-1}$	0.551	0.589	-0.004	0.002
	(0.673)	(2.199)	(-0.194)	(0.069)
$JY\Delta\hat{v}$	0.025	0.003	-0.057	-0.008
	(0.337)	(0.132)	(-4.514)	(-2.973)
$JY\Delta\hat{v}_{-1}$	0.016	0.008	-0.030	-0.002
	(0.201)	(0.280)	(-2.194)	(-0.531)
$JY\Delta\hat{v}_{-2}$	0.015	0.004	0.004	-0.002
	(0.210)	(0.160)	(0.330)	(-0.896)

Significant effects are found for both the forward-looking and lagged, backward-looking measures of volatility. The effects on the expected change in the exchange rate are not statistically significant but are generally large in magnitude compared to those of the other (even significant) variables. This in turn may suggest that the impact of changes in volatility on changes in the expected change is rather sizable, but simply rather unstable over time and thus imprecisely estimated. This is very similar to the error-correction results on the expected change in the exchange rate for many of the models which included volatility, where the adjustment was quite large, though in some instances not significant.

The most striking result is that the short-run effect of volatility appears to act almost wholly through the variables associated with the gap (the gap itself and the inflation rates). If we exclude the gap and inflation rates, we observe almost no significant impacts of volatility on the other variables, those associated with the premium, and none in fact for two of the three samples. Thus it seems it is important to incorporate the gap effect, not only to establish cointegration and understand movements in the level of the premium, but also to understand the connection of volatility to the premium.

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