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Rationality in the Present-Value Model of Stock Prices:
Fundamentals, Psychology, and Structural Change

Nicholas J. Mangee
nicholas.mangee@armstrong.edu
Armstrong Atlantic State University
INET Center on IKE

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Rationality in the Present-Value Model of Stock Prices: Fundamentals, Psychology, and Structural Change

The present-value model of stock prices is a workhorse in financial economics.¹ The model relates today's price of a stock (or a basket of stocks) to the market's forecasts of next-period's price and dividend, appropriately discounted. This specification formalizes the commonsense notion that a rise in the market's price or dividend forecast will lead to a rise in the stock price today. The time series implications of the model depend on how the market's price and dividend forecasts are assumed to unfold over time. The standard solution makes use of the rational expectations hypothesis (REH). This hypothesis assumes that market participants can fully foresee when and how their understanding of the price and dividend process changes over time. Economists usually assume that this understanding not only does not change in unexpected ways, but does not change at all.² A typical solution assumes a constant discount factor and a dividend process that follows a random walk with constant drift. The resulting REH-based present-value model implies that stock-price movements relative to the known constant drift occur only because of random disturbances to the dividend. According to this model, stock prices fluctuate so as to maintain a constant price-dividend ratio.

It is difficult to reconcile this implication with the tendency of the Standard and Poor's Composite (S&P 500) and other major stock price indexes to undergo wide and persistent swings away from and back toward benchmark values based on historical averages of price-dividend or price-earnings ratios. Even when researchers use the *ex post* values of future dividends to measure the benchmark, prices swings away from this benchmark are much too persistent (Shiller, 1981, 2003). Moreover, researchers' difficulty in finding a cointegrating relationship with dividends or earnings suggests that stock-price fluctuations do not depend at all on these fundamentals.³

Behavioral finance has interpreted this evidence as a failure of REH to account for how market participants actually make decisions. But, instead of searching for a different standard of rationality, researchers concluded that market participants are irrational. In behavioral-finance models, asset prices are largely driven not by fundamentals such as dividends or earnings, but by bubbles that arise because participants fall prey to psychological biases, emotions, and momentum trading.⁴

In this paper, we advance an alternative explanation of the empirical difficulties of the REH-based present-value model. The problem is not that market participants are irrational. Rather, Frydman and Goldberg (2013b) show that REH models are compatible with rational forecasting only in "markets" in which participants can fully foresee when and how their understanding of the price process might change. They show that REH models are in effect abstractions of rational decision making in markets in which knowledge does not grow.⁵ But, in real-world markets knowledge does grow, for example, participants at times find new

¹For the seminal studies, see Samuleson (1965, 1973).

²For a formal definition of "participants' understanding" and how contemporary macroeconomics and finance models abstract from its unexpected changes, see Frydman and Goldberg (2013a, 2013b).

³For example, see Campbell and Shiller (1987, 1988), Timmermann (1995), and Kanas (2005).

⁴See DeLong et al. (1990), Brunnermeier (2001), and references therein.

⁵This statement is trivially true for the vast majority of REH models, which are time invariant and thus assume that the market's understanding of the price process never changes. Sometimes, REH theorists allow

ways to understand the consequences of economic policy on the structure of the economy. In those markets, REH models represent decision making by individuals who forgo obvious profit opportunities and thus are grossly irrational.

Frydman and Goldberg (2013b) advance an analog to REH that is relevant for building models that could be compatible with rational decision making in markets in which knowledge grows. According to this analog, which they call the contingent expectations hypothesis (CEH), rational market participants understand that they live in a world in which the knowledge that underpins the market's forecast is contingent: it changes at times and in ways that no one can fully foresee.

In section 1, we show how CEH affects the solution of the present-value model. In contrast to its REH counterpart, this solution recognizes that market participants forecast over finite horizons. Iterating the model forward over a finite horizon enables us to express today's price in terms of the market's forecasts of the stream of future dividends and interest rates (which we use as discount rates) and the final price. Unlike the REH solution, our CEH solution recognizes that market participants rely on a much broader information set than current dividends to forecast future dividends, interest rates, and stock prices.

Indeed, because rational participants' recognize that their knowledge is imperfect and contingent, they do not rely on fundamentals and calculation alone. They must also rely on psychological and social factors in forecasting dividends, interest rates, and stock prices. Psychological considerations, such as the confidence that an individual places in her current strategy or the intuition she has about possible structural change, are essential in rational decision making. As Keynes (1936, p. 162) put it,

We are merely reminding ourselves that human decisions affecting the future, whether personal or political or economic, cannot depend on strict mathematical expectation, since the basis for making such calculations does not exist; and...that our rational selves [are] choosing between alternatives as best as we are able, calculating where we can, but often falling back for our motive on whim or sentiment or chance.

We formalize this observation by relating our representation of the market's forecast to an index variable that we call "psychology." This variable represents the influence of psychological considerations on participants' forecasts.

Keynes (1936, p. 162) also understood that rational participants would also pay close attention to the "political and social atmosphere" in forecasting the market. In fact, there is much anecdotal evidence (for example, from business journalists) that news on a wide range of economic, political, and natural developments move markets, including central bank announcements, the Congressional battle over fiscal policy, geo-political instability and war, and natural disasters. This news is part of the social context within which rational

for the price process to change, for example, because of shifts in economic policy. But, in order to obtain a probabilistic representation of the price process, these models must specify in probabilistic terms all such structural change in advance. Doing so represents any change in the market's understanding in a way that can be fully anticipated. Such representations, therefore, assume away all growth of participants' knowledge. This argument follows from Popper's (1957, xii) proposition: "If there is such a thing as growing human knowledge, we cannot anticipate today what we shall only know tomorrow."

participants must make their trading decision. Like with psychology, we formalize this observation by relating our representation of the market’s forecast to an index variable that we call “social context.”

Unlike with REH, price fluctuations in our CEH model stem in part from changes in how the market forecasts dividends, interest rates, and stock prices. To generate implications for time series data, the model must specify how participants’ forecasting strategies unfold over time. CEH implies that there are stretches of time of unpredictable duration in which market participants largely keep their forecasting strategies largely unchanged or revise them only moderately. This assumption gives rise to a piece-wise linear specification for stock prices. We show how each linear piece is characterized by a distinct cointegrating relationship between the stock price, dividend, interest rate, social context, and psychology. The timing of points of structural change, which determine when a linear piece begins and ends, and how exactly the dividend, interest rate, social context, and psychology enter the temporary cointegrating relationships is left open in the model.

We estimate our CEH present-value model for the S&P 500 price index using a cointegrating VAR approach.⁶ In doing so, we need to address two issues: locating points of structural change in the data and measuring our social context and psychology variables. We address the first issue by appealing to Frydman and Goldberg’s (2013b) model of asset price swings and risk, which suggests that major break points are likely to be found at major turning points in the data. In order to measure our social context and psychology variables, we rely on a novel data set developed by Mangee (2011), which is based on textual information contained in daily market “wrap” stories prepared by *Bloomberg News* at the end of each trading day. We discuss these data in section 2.

Section 3 presents the results of our empirical analysis. Like other empirical studies, we are unable to reject the null of no cointegration between stock prices, earnings, and interest rates when we ignore structural change and the influence of psychology and the social context in the model. Allowing for mean shifts in the stock-price relations leads to a marginal rejection of the null of no cointegration. It is only when we allow for structural change in the cointegrating space *and* account for the influence of psychology and the social context that we find strong evidence of a cointegrating relation in the data. Moreover, we find that all of our informational variables enter the identified cointegrating space with significant coefficients that are largely consistent in sign with our CEH model’s predictions. Overall, our results indicate that the empirical difficulties that the present-value model of stock prices have encountered in the literature are not due to the presence of irrational traders, but to the inability of REH to represent the forecasting behavior of rational participants.

1 Rationality in the Present-Value Model

The present-value model of stock prices can be expressed as follows:

$$P_t = \frac{\mathcal{F}_t [P_{t+1} + D_{t+1}]}{1 + i_t} \tag{1}$$

⁶See Johansen (1996) and Juselius (2006) for book-length treatments of the CVAR model.

where P_t and D_t denote the time- t price of a stock or a basket of stocks and the dividend, respectively, i_t is the nominal interest rate, and the $\mathcal{F}_t[\cdot]$ denotes a point forecast that represents an aggregate of participants’—the market’s—time- t forecasts of prices and dividends. Under REH, these forecasts are represented by the mathematical expectation that is implied by equation (1), a given stochastic process for dividends and interest rates, and the assumption that the market’s expectations of prices, dividends, and interest rates are correct on average at every point in time.

The REH solution presumes that the market can fully anticipate how it will understand the price process in all future periods. To see this, we iterate equation (1) forward one period, take time- t expectations, and substitute the result back into the equation, yielding:

$$P_t = \frac{E_t[D_{t+1}]}{1 + i_t} + \frac{E_t E_{t+1}[P_{t+2} + D_{t+2}]}{(1 + i_t)(1 + E_t[i_{t+1}])} \quad (2)$$

where $E_t[\cdot]$ is the expectations operator. The expression shows that the economist must represent the market’s time- t forecast of how it will forecast prices and dividends at $t + 1$. The standard REH solution assumes time-invariant processes for the dividend and interest rate, thereby representing the market’s understanding of these and the price process to be the same at every point in time.⁷ Imposing REH, therefore, enables one to apply the law of iterated expectations and set $E_t E_{t+1}[P_{t+2} + D_{t+2}] = E_t[P_{t+2} + D_{t+2}]$. Carrying out the forward iteration to infinity and assuming a constant discount factor and a dividend process that follows a random walk with a drift, implies a stable cointegrating relationship that involves a constant price-dividend ratio.

1.1 The Contingent Expectations Hypothesis

By contrast, CEH builds on Popper’s proposition, which we slightly paraphrase as follows:

If there is such a thing as growing human knowledge, then no individual, such as an economist or a market participant, or group of individuals, such as market participants in the aggregate, can anticipate today what they shall only know tomorrow. (Popper, 1957, xii)

This insight underpins the first of CEH’s two pillars, called the principle of contingent knowledge: to be relevant for representing how a profit-seeking individual understands and forecasts the process driving market outcomes, a model should recognize that this process is contingent, that is, it is subject to change at times and in ways that cannot be fully anticipated. This principle implies that for an economic model to be compatible with rational forecasting in real world markets, in which knowledge grows, it must be partly open to unanticipated *structural* change.⁸

⁷REH models sometimes allow for change in the process underpinning the causal variables, but when they do, they assume all structural change can be fully anticipated in probabilistic terms.

⁸These models constrain change in their structures over time; but, conditional on any one of the causal structures that is implied by a model at a given point in time, they do not specify in advance the *exact* structures that may be needed to represent the market process at any other point in time. See Frydman and Goldberg (2013b) for a mathematical example and further discussion

Frydman and Goldberg’s (2007) imperfect knowledge economics (IKE) provides an approach for building such models. As we show below, an IKE model stops short of specifying fully in advance how the market’s understanding of the price, dividend, and interest rate process unfolds over time. It does so by imposing qualitative and contingent restrictions on this structural change.

But not all IKE models are compatible with individual rationality. This is because IKE models imply regularities in time-series data, and thus, its representations of market participants’ forecasting must be compatible with these regularities. Otherwise, they would presume that participants forego profit opportunities. This reasoning underpins the CEH’s second pillar, called the principle of internal coherence: representations of participants’ forecasting cannot imply regularities in time-series data that can conflict with the regularities that are implied by the model’s account of aggregate outcomes. Like internal consistency in REH models, this principle connects an IKE model’s representation of forecasting to the specifications of its other components. It also implies restrictions on structural change in a model.

1.2 A CEH Solution

In solving the model with CEH, we rely on the following log-linear approximation:

$$p_t = \mathcal{F}_t [p_{t+1} + d_{t+1}] - i_t \quad (3)$$

where p_t denotes the logarithm of P_t and d_{t+1} is the dividend yield, D_{t+1}/P_{t+1} .⁹ This semi-reduced-form equation formalizes economists’ understanding of the price process. As with REH, CEH uses this understanding in representing the market’s price forecast. This enables us to iterate equation (3) forward a finite number of periods, take time- t forecasts, and use the results to represent the market’s forecasts in future periods. The finite number of periods recognizes that market participants forecast over finite horizons. For simplicity, we carry out this procedure one period, resulting in:

$$p_t = \mathcal{F}_t \mathcal{F}_{t+1} [p_{t+2} + d_{t+2}] - \mathcal{F}_t [d_{t+1} - i_{t+1}] - i_t \quad (4)$$

Like before, we must represent the market’s time- t forecast of how it will forecast prices and dividends at $t + 1$. If we had iterated forward more than one period, we would need to represent forecasts of forecasts of forecasts. CEH recognizes that no one can anticipate fully today how the market will understand the price, dividend, and interest rate process in future periods. Nonetheless, our CEH model must provide some representation of the time- t knowledge that is used to forecast the market’s future understanding and market outcomes.

To this end, we portray the market’s understanding in future periods as linear relationships involving a set of causal variables:

$$p_t = \beta_t^2 x_t^2 + \beta_t^1 x_t^1 - i_t \quad (5)$$

where the vector x_t^j represents the union of casual variables that market participants use in forecasting the market’s understanding and outcomes in period $t + i$ and the vector β_t^j

⁹Equation (3) is derived by setting to zero the *ex post* return on holding stocks net of the cost of capital, taking log approximations, and passing the forecast operator through the resulting expression.

represents the weights that they attach to these variables in making their forecasts. As with the REH specification in (2), we use the specification in (5) to represent the market process at every point in time. However, our CEH model recognizes that the market’s understanding changes at times and in ways that cannot be fully foreseen: it allows its representation of the market’s understanding to change ($\beta_t^2 x_t^2 + \beta_t^1 x_t^1$ can differ from $\beta_{t+1}^2 x_{t+1}^2 + \beta_{t+1}^1 x_{t+1}^1$) in ways that are left partly open.

Equations (3) and (5) imply that we can express the market’s forecasts of next-period’s price and dividend as follows:

$$\mathcal{F}_t [p_{t+1} + d_{t+1}] = \beta_t x_t \quad (6)$$

where the vector x_t is the union of variables in the x_t^j ’s and the elements of β_t are summations of the β_t^j ’s for each variable. In specifying this representation, our CEH solution makes use of the understanding that is formalized in expressions (3) and (4). This understanding implies that higher dividends or lower interest rates leads to higher prices. We therefore include current dividends and the interest rate in x_t and assume that the weights that are attached to these variables, β_t^d and β_t^i , are positive and negative, respectively.¹⁰

1.2.1 The Social Context and Psychology

Rational market participants’ recognize that their knowledge is imperfect and contingent and thus they do not rely on observable fundamentals and calculation alone. They also pay close attention to the “political and social atmosphere” within which they must make their trading decisions. It is this atmosphere that provides a way for them to understand how other participants’ might be thinking about the future and thus to forecast the market. In discussing the importance of the social context for Keynes’ thinking about how rational individuals make decisions, Dow (2013, p. 114, emphasis added) writes,

Individuality or agency allows for individual choice as to whether or not to follow social convention. But sociality means that social-conventional judgment provides the norm, such that expectations are formed *interdependently* with expectations in the market. This non-deterministic social interactionism is a key ingredient of Keynes’s... view of the economic system.

Not surprisingly, Mangee’s (2011) *Bloomberg News* data (see Section 2) shows that news on a wide range of economic, political, and natural developments move stock markets. Our CEH representation formalizes the influence of these considerations on the market’s forecast by an index variable that we call “social context” and denote by c_t .

The imperfection and contingent nature of knowledge also implies that rational market participants must also rely on psychological considerations, such as the confidence that an individual places in her current strategy or the intuition she has about possible structural change, in forecasting market outcomes. As Keynes’ (1936, p.148) put it, the “state of

¹⁰We are assuming that the market interprets higher dividends or lower interest rates in the current period a reason to forecast further movements of these variables in the same direction. Internal coherence would imply such forecasts if the contingent processes for dividends and interest rates were assumed to involve persistent trends. For a treatment on internal coherence’s implications concerning the market’s understanding of the processes driving the causal variables, see Frydman and Goldberg (2013c).

confidence...[is a factor to which] practical men always pay the closest and most anxious attention.” Keynes (1936) is rather explicit about how psychological considerations, such as confidence and optimism, exert their impact; they do so through the manner in which participants interpret and alter their “knowledge of the facts...which will influence the...existing market valuation” (p. 152): the “state of confidence,” does not

[individuals’ expectations do not] solely depend...on the most probable forecast...[but] on the confidence with which we make this forecast – on how highly we rate the likelihood of our best forecast turning out quite wrong. If we expect large changes but are very uncertain as to what precise form these changes will take, then our confidence will be weak (Keynes, 1936, p.148).

Our CEH representation formalizes the influence of psychological considerations on the market’s forecast by an index variable that we call “psychology” and denote by s_t . How s_t and the social context are assumed to influence participants’ forecasts depends on whether they forecast rising prices and thus hold long positions in stocks or they forecast falling prices and hold short positions.

1.2.2 Representing Bulls’ and Bears’ Forecasts

Each market participant formulates a forecasting strategy that reflects her own knowledge, intuition, and confidence about which factors are relevant and how each one should be interpreted in thinking about the future. At each moment in time, participants’ forecasts are not only diverse, but they involve differing predictions about the direction of change. CEH provides a way to represent the diversity of participants’ forecasting strategies without presuming that they are irrational and forego profit opportunities.

In this paper, we suppose that the forecasting strategies of the group of bulls and bears concerning next-period’s prices and dividends can be represented as follows:

$$\mathcal{F}_t^j [p_{t+1} + d_{t+1}] = \beta_t^j x_t^j \tag{7}$$

where $x_t^j = [\rho_t^j \ e_t \ i_t \ s_t^j \ c_t^j]$, e_t denotes the logarithm of corporate earnings, which research shows is a good predictor of future dividends, ρ_t^j is a mean value that reflects the change from dividends to earnings in the informations set, and $j = L, S$ denotes the forecast of the group of bulls and bears, respectively.

Rising confidence on the part of bulls and bears has opposite effects on the price of stocks. We represent these opposite effects by assuming that s_t^j enters bulls’ forecasting strategies with a positive weight, whereas for bears, it enters with a negative weight. Consequently, a rise in the confidence and optimism of bulls (bears) in the model, that is, a rise in s_t^L (s_t^S), leads them to raise (lower) their forecasts of p_{t+1} and bid up (down) today’s prices.

A market participant’s confidence, of course, depends in part on purely psychological elements, and it is impossible for anyone to look directly into the psyches of other participants. How, then, might participants “pay the closest... attention” to the state of confidence in the market, let alone forecast its future unfolding?

Relating the state of confidence to the uncertainty of knowledge suggests that it is connected to the fundamental factors that participants use to forecast market outcomes. For

example, rising company earnings or overall economic activity may lead the confidence of bulls to increase and that of bears to decrease. Indeed, five lines after Keynes points out that speculation entails forecasting “what the market will value [an asset] at, under the influence of mass psychology, three months or a year hence,” he reveals how to pay attention to the state of confidence and play the beauty contest: “Thus, the professional investor is forced to concern himself with the anticipation of impending changes, in the news. . . of the kind by which experience shows that the mass psychology of the market is most influenced” (p. 155). In the empirical analysis of section 3, therefore, we would expect to find that our *Bloomberg News* measure of psychology is related to fundamental factors.¹¹

We define our social context variable for bulls and bears in similar fashion. Positive news about economic and political developments that is viewed by bulls as positive is often interpreted as negative news by bears. We again represent these opposite effects by assuming that c_t^j enters bulls’ forecasting strategies with a positive weight, whereas for bears, it enters with a negative weight.

It is, of course, difficult to anticipate which pieces of news will influence the market, let alone the nature and size of their separate impacts on forecasting. The economic, political, and natural developments that characterize the social context unfold over time in ways that no one can fully anticipate their timing or nature, or their impact on the market’s forecast. Moreover, no one would expect bulls’ and bears’ confidence and optimism to be connected to fundamental considerations in any fixed way.

As we make clear in section 2, Manglee’s (2011) textual data enable us to measure psychological and social-context considerations without assuming that the composition of the factors that underpin movements in s_t^j and c_t^j remains fixed over time.

1.2.3 Representing the Market’s Forecast

Bloomberg News reports on the impact of psychological and social-context considerations for the market as a whole and not for the trading decisions of the group of bulls and bears separately. We thus formulate our CEH representation in terms of an aggregate psychology and social context variable:

$$\mathcal{F}_t^j [p_{t+1} + d_{t+1}] = \beta_t x_t \tag{8}$$

where $x_t = [\rho_t \ e_t \ i_t \ s_t \ c_t]$, s_t and c_t represent aggregate measures of the psychological and other fundamental considerations that influence the market’s forecasting, ρ is an aggregate mean, and β_t is a vector of aggregate weights.

We note that in moving to the formulation in (8), our aggregate measures of psychology and the social context take into account that s_t^j and c_t^j enter bulls’ and bears’ forecasting strategies with weights whose signs differ. There are thus two reasons why the aggregate s_t and c_t variables may rise and two reasons why they may fall. For example, a rise in one or both of these measures would occur if news was interpreted by bulls as positive or bears as negative, leading one group to increase and the other to decrease their confidence and optimism or their assessment of social context.

¹¹*Bloomberg News*’ reporting bears this claim out. Manglee (2011) finds that mentions of confidence and other psychological considerations driving U.S. equity prices in market wrap stories are almost invariably connected to fundamental developments.

Equations (8) and (3) imply the following model of stock prices:

$$p_t = \rho_t + \beta_t^d d_t - \beta_t^i i_t + \beta_t^s s_t + \beta_t^c c_t \quad (9)$$

where the algebraic signs of ρ_t and the β_t -weights have been made explicit. This representation implies that there are two key factors that underpin the evolution of stock prices: revisions of the market’s forecasting strategy—changes in β_t —and movements of the causal factors. In order to derive time series implications from the model, we need to represent how participants might revise their forecasting strategies, that is, we need to restrict how β_t changes over time.

1.2.4 Revisions of Forecasting Strategies and Time Series Implications

Unlike REH, the CEH constraints that we impose on this change recognize that no one, including economists, can fully anticipate when and how market participants might decide to revise their forecasting strategies. In modeling this change, our CEH representation again appeals to Keynes’s (1936) account of asset markets. In using their “knowledge of the facts” to form forecasts, participants

“fall back on what is, in truth, a convention. . . [which] lies in assuming that the existing state of affairs will continue indefinitely, except in so far as we have specific reasons to expect a change.” (Keynes, 1936, p. 152)¹²

This insight suggests that market participants tend to stick with a forecasting strategy for stretches of time. Indeed, it is often unclear whether one should alter her strategy. A quarter or two of poor forecasting performance may be the result of random events rather than an indication of a failing strategy. So, unless an individual has “specific reasons to expect a change” in the market, she may leave her current strategy unaltered – even if its performance begins to flag over several periods. Moreover, even armed with “specific reasons to expect a change,” it is entirely unclear what new forecasting strategy, if any, she should adopt.

Our CEH representation of structural change in (8) formalizes this insight with qualitative and contingent constraints that Frydman and Goldberg (2007, 2013b) call “guardedly moderate revisions”: there are stretches of time during which participants either maintain their strategies or revise them gradually. It is clear from equation (8) that any stretch of time in which market participants in the aggregate kept their forecasting strategies unchanged would involve a temporary but stable equilibrium relationship between stock prices, fundamentals, and psychology. Moreover, if a stretch of time also involved some points at which revisions of strategies were sufficiently moderate, the model would continue to imply that the sign of each of the weights that were attached to the e_t , i_t , s_t , and c_t variables would remain unchanged.¹³

Consequently, if we were to run a regression of stock prices on these variables during a stretch of time in which revisions were guardedly moderate, we would expect to find

¹²By “existing state of affairs,” Keynes means “knowledge of the facts.”

¹³The conditions that are needed for the model to imply these qualitative relationships is $|\beta_{t-1}^h \Delta x_t^h| > |\Delta \beta_t^h x_t^h|$, where $|\cdot|$ denotes an absolute value and $h = e, i, s, \text{ or } c$. For more discussion on such guardedly moderate constraints and how they are implied by internal coherence, see Frydman and Goldberg (2013a).

an approximate cointegrating relationship, given that these variables are often modeled as having stochastic trends. Moreover, the model would predict that the causal variables would enter this temporary cointegrating relationship with weights whose signs were consistent with those in equation (9).

However, although market participants have a tendency to maintain their strategies or revise them gradually, this qualitative regularity is contingent: it manifests itself at times and in ways that no one can fully foresee. There are occasions when price movements or news about economic and political developments lead participants to revise their forecasting strategies in non-moderate ways. Such revisions can have a dramatic impact on the price process and spell the end of any stretch of time that was characterized by a temporary cointegrating relationship between prices, fundamentals, and psychology.

Frydman and Goldberg (2007, 2013b,c) and Frydman et al. (2013) show that the CEH model of stock prices can account for Shiller’s (1981) and Campbell and Shiller’ (1987, 1988) finding that stock-price fluctuations are too persistent to be rationalized with standard REH models. Two assumptions are needed: market participants’ tendency to stick with their current strategies or revise them gradually is pronounced and the fundamental factors on which they form their forecasts entail persistent trends.

In this paper, we focus on estimating temporary cointegrating relationships in the data. To this end, we approximate the contingent change in the price process with a piece-wise linear error-correction model.

1.3 A CEH Econometric Specification

Equation (9) implies the following temporary error-correction specification for each stretch of time in which participants largely revise their forecasting strategies in guardedly moderate ways:

$$\Delta p_t = \alpha [p_t - \beta^j x_t - \rho^j] + \beta^j \Delta x_t + \varepsilon_t \quad (10)$$

where $\alpha = -1$ and $j = 1, 2..$ denotes distinct stretches of time in the data for which the models’ parameters are, in a statistical sense, relatively stable. During each of these stretches, the model implies a temporary cointegrating vector that embodies the qualitative relationships in equation (8).

One of the issues in estimating (10) is that our CEH model does not fully specify in advance when one linear piece of the data ends and another one begins; such break points stem from non-moderate revisions of participants’ forecasting strategies, which occur at times and in ways that no one can fully foresee. In Frydman and Goldberg (2007) and Mangee (2011) we address this issue by relying on recursive structural change procedures that enabled us to refrain from specifying the break points *a priori*.

In this paper, we rely on a different procedure, one that entails *a priori* considerations. One of the implications of Frydman and Goldberg’s (2007, 2013b) IKE model is that points of non-moderate revisions and thus of structural change are likely to be found at major turning points in the market, where a long swing in asset prices away from commonly used benchmark values ends and a sustained counter-movement back towards benchmark values sets in. As we report in Section 3, we in fact find that these points in the data are characterized by

structural change. We simplify our structural change analysis by testing for mean shifts in the cointegrating space (that is, in ρ^j), while assuming the other parameters of the temporary cointegrating vectors are, in a statistical sense, relatively stable.¹⁴

2 *Bloomberg News: Fundamentals and Psychology*

The CEH present-value model implies that rational market participants must rely on psychological considerations and the social context in forecasting market outcomes. To measure these factors, we use a novel dataset developed by Mangee (2011), which is based on information contained in *Bloomberg News*' end-of-the-day equity market "wraps" for the period 1993-2009.¹⁵ In writing the reports, *Bloomberg's* journalists rely on contacts with 100-200 equity fund managers and other professional market participants. These stories provide a window into the decision-making of those whose trading ultimately determines prices.

The social context refers to a broad set of factors, in addition to conventional data series such as earnings and interest rates, that market participants may use in forecasting market outcomes. There are many developments that participants in *Bloomberg's* wrap stories consider relevant, including statements by the Federal Reserve's chairman, political instability, natural disasters, or SEC regulatory policy. Table 1 lists the factors that were included in our social context variable.¹⁶

To quantify the information on the social context in the market wrap stories, we tabulated monthly averages of the frequencies with which such factors were mentioned. There are, however, both positive and negative social context "events." In constructing our aggregate measure, we define positive events as those that led to a rise in the stock price, and negative events as those leading to a decline in price. Our aggregate social context measure was constructed by subtracting the monthly frequency of "negative" events from the monthly frequency of "positive" events.

Bloomberg market wrap stories also contain evidence that psychological considerations play a role in driving the market.¹⁷ Table 2 lists the psychological considerations that *Bloomberg* journalists reported were important drivers of the market. The methodology used to measure our aggregate "psychology" variable is identical to that of our social context variable.

¹⁴Hendry (2000) demonstrates that mean shifts in the cointegrating space are generally easier to detect than instability in the dynamic components of the model.

¹⁵For a detailed treatment of the construction of the *Bloomberg* dataset, and of its benefits and limitations, see Mangee (2011).

¹⁶For an example of the information behind these data, consider the market wrap on November 30, 2007: "U.S. stocks rose, capping the best weekly gain since March, after Federal Reserve Chairman Ben S. Bernanke signaled he may cut interest rates...Saying in a speech after the market closed yesterday that the economic outlook has become more uncertain, requiring the central bank to be 'exceptionally alert and flexible.'"

¹⁷Consider the report from March 2, 2009: "You have got a lot of fear going into earnings," said John Nichol, who manages \$1 billion in Pittsburgh, including the Federated Equity Income Fund, which has beaten 74 percent of its peers over the past five years." Conversely, the wrap for April 21, 2009, read as follows: "IBM earnings are extremely positive," said Howard Cornblue, a money manager from Pilgrim Investments, which oversees \$7 billion. "This will give confidence and stability to the market."

3 The Cointegrated VAR Model

This paper uses Johansen's (1996) Gaussian Maximum Likelihood Estimation procedure to test the temporary equilibrium relationship between stock prices, earnings, interest rates, psychology, and the social context. Consider a p^{th} -order vector time series, x_t where $x_t' = [p_t, e_t, i_t, s_t, c_t]$. The vector x_t is assumed to follow the VAR process,

$$\Pi(L)x_t = \Phi_t + \epsilon_t \quad (11)$$

where $\Pi(L)$ is a matrix polynomial of order k and L denotes the lag operator whereby $\Pi(0) = I$ and Φ_t contains the deterministic terms and ϵ_t is a vector I.I.D. error term. Equation (11) can be transformed into,

$$\Delta x_t = \Pi x_{t-1} + \sum_{j=1}^{k-1} \Gamma_j \Delta x_{t-j} + \Phi D_t + \epsilon_t \quad (12)$$

where,

$$\Pi = \alpha\beta' \quad (13)$$

The Π term is the long-run matrix where α and β are $p \times r$ matrices and r , the reduced rank of Π , denotes the number of stationary equilibrium relationships based on the unrestricted VAR model. In the presence of unit roots in x_t , the reduced rank condition, $r \leq p$ can be imposed on the longer-run matrix, $\Pi = \alpha\beta$, testing for the existence of at least one cointegrating vector in x_t . The β matrix represents the temporary cointegrating relationships, given the system of equations in the unrestricted VAR model, while the α matrix represents the error-correction terms and their corresponding speed of adjustment toward temporary equilibrium.

The existence of a cointegrating relationship can be tested in equation (12) by applying the MLE procedure and imposing a reduced rank on Π . Regressing Δx_t and x_{t-1} on $\Delta x_{t-1}, \Delta x_{t-2}, \dots, \Delta x_{t-k+1}$ and D_t generates the residuals R_{0t} and R_{1t} , respectively. Defining the product moments as $S_{ij} = T^{-1} \sum_{t=1}^T R_{it} R_{jt}'$ and solving the eigenvalue problem yields, $|\lambda S_{11} - S_{10} S_{00}^{-1} S_{01}| = 0$. The trace-test statistic represents the likelihood ratio (LR) test of the null hypothesis of at least r cointegrating relationships in x_t , and takes the form $-T \sum_{i=r+1}^p \ln(1 - \hat{\lambda}_i)$, where $\hat{\lambda}_i$ represents the r -largest non-zero eigenvalues.

3.1 Empirical Analysis

The data in the information set is of monthly frequency over the sample period January 1993-December 2007.¹⁸ Stock-market prices are measured by the Standard and Poor's 500 Composite Index, as are aggregate earnings.¹⁹ There is much evidence suggesting that earnings predict future dividends and that the former are a more appropriate measure than

¹⁸The sample period's end was chosen because the massive shock to earnings from the global financial crisis led to a poorly specified empirical model.

¹⁹S&P500 prices and earnings are measured in logarithmic form. Data are obtained from Robert Shiller's online database at www.econ.yale.edu/~shiller/.

the latter of cash flows accruing to equity ownership.²⁰ Data on the 3-month U.S. Treasury bill is used for the interest-rate series, and social context and psychology are as described in Section 2. The empirical results are based on the I(1) analytical framework, and Figure 1 provides graphical representations of the data.²¹ To test the implications of the CEH-based present-value relationship, a “nested-models” methodology is employed as follows.

The CEH-based model (Model 3) includes the information vector $x'_t = [p_t, e_t, i_t, s_t, c_t]$ and the deterministic terms in ΦD_t . Tests for mean shifts at major reversals in the price-earnings valuation ratio (1999:12 and 2003:02), or breaks, in the cointegrating space are highly significant. We allow for these shifts in our empirical model. We reject the joint restriction that both mean shifts can be excluded from the cointegrating space, $D_s 1999 : 12 = D_s 2003 : 02 = 0$, $\chi^2(6) = 11.457[0.075]$.²² Ignoring parameter instability leads to a poorly specified model that invalidates standard statistical inference.

We then compare empirical analyses across sub-models within the CVAR framework.²³ Model 2 examines the traditional present-value relationship, where $x'_t = [p_t, e_t, i_t]$, with the inclusion of mean shifts and other deterministic terms in ΦD_t . Finally, a baseline traditional present-value model (Model 1) is brought to the data, with the information set containing only $x'_t = [p_t, e_t, i_t]$ without social context, psychology, and ΦD_t . Assessing how much the inclusion of deterministic terms and social context and psychology affect the underlying present-value relationships yields a more robust and rich framework for shedding light not only on the empirical failures of the traditional model of the present-value stock-price relationship, but also on the implications of the CEH-based model.

3.2 Testing for Temporary Cointegrating Relations

The first stage of analysis formally tests for the existence of at least one cointegrating (equilibrium) relationship within the information set in question. As discussed in Section 3, the trace test delivers p-values based on the likelihood-ratio test as the primary tool for determining the rank of the Π -matrix. Table 3 reports trace tests and likelihood-ratio-generated p-values for the three models under consideration.

Trace tests for the baseline model of stock prices, earnings, and interest rates fail to reject the null hypothesis of no cointegration with a p-value = [0.119], which is consistent with findings in the literature. If mean shifts and other deterministic terms are included, there is evidence of one equilibrium relationship where we are able to reject the null hypothesis of no cointegration at the 95% level but not at 99%. This tentative finding of $r = 1$ is an improvement over Model 1. The CEH-based Model 3 finds strong evidence of three equilibrium relationships in the data. Moreover, the joint restriction that s_t and c_t can both be excluded from the cointegrating space is strongly rejected with a p-value=[0.000]. Taken together, these results suggest that only when structural change, social context, and

²⁰See, for example, Marsh and Merton (1987) and Campbell (2000), and references therein.

²¹Stock prices, earnings, interest rates, and psychology each contain at least one unit root. Social context for the sample 1993:01-2009:12 is also I(1), and is near-I(1) for the sample ending in 2007:12. Signs of I(2)-ness in the data are left for future research.

²²These breakpoints were also identified in Mangee (2011) using a sequential recursive test for detecting and identifying points of structural change.

²³All three models include a lag length equal to 2 and include corrections for seasonality in the data.

psychological considerations are included in the model is there strong evidence of a temporary equilibrium relationship, as implied by the CEH-based present-value model.

Table 4 reports across-model results for multivariate residual diagnostic tests. A well specified model is required for valid statistical inference of the trace test statistics. For this reason, serial correlation is of particular concern. Tests for residual serial correlation, normality, and autoregressive conditional heteroskedasticity (ARCH) show that the null hypothesis of no serial correlation can be accepted for all three models, increasing our confidence in the trace-test results.²⁴ Residual properties for Model 2 are a slight improvement over Model 1, as second-order ARCH effects decrease. Model 3 displays better residual properties, accounting for both first- and second-order ARCH effects.²⁵

3.3 Temporary Cointegrating Relations: Estimation and Identification

Empirical evidence from the previous section yields consistent results in favor of the predictions of the CEH-based model. The remainder of the statistical analysis examines in greater detail the inner dynamics of the information set guided by the model’s predictions. With a cointegration rank $r = 3$, we have $p - r = 2$ common stochastic trends, where $p = 5$ refers to the number of variables in the system. Evidence on the source of common “driving” forces in the cointegrating relationships comes from loading each variable onto $\alpha'_{\perp 1}$ and $\alpha'_{\perp 2}$. Table 5 shows that the common stochastic trends are cumulated shocks to earnings and interest rates. This suggests that earnings and interest rates are the driving forces underlying the equilibrium relationships. This also supports the finding that interest rates are weakly exogenous to the system.

The restrictions on the identified equilibrium structure are accepted with a p-value= [0.582]. The $\hat{\beta}'_1$ -vector appears to be a relationship between social context, interest rates, earnings, and psychology with small but strongly significant effects from stock prices, where: $c_t \approx i_t + 0.26(e_t + s_t) - 0.90p_t + \dots$. Earnings and psychology enter with the hypothesized positive signs. However, stock prices enter with a negative coefficient. Furthermore, the first β relationship is significantly equilibrium-correcting in terms of social context.

The $\hat{\beta}'_2$ -vector appears to be a relationship between stock prices, earnings, psychology, social context, and strong effects from interest rates where $e_t \approx .595p_t + 0.985s_t + 0.405c_t - 5.128i_t + \dots$. The positive coefficients on stock prices, psychology, and social context, and the negative sign on interest rates, are all as hypothesized. The cointegrating relationship is found to be significantly equilibrium-correcting in Δp_t and Δe_t . The $\hat{\beta}'_3$ -vector appears to display a relationship in terms of social context, with significant effects from psychology and stock prices, where $c_t \approx 0.114p_t + s_t + \dots$. As we would expect given our CEH-based IKE model, the social context, stock prices, and psychology all share positive relations. The

²⁴Univariate tests for skewness and kurtosis were also conducted. Models 1 and 2 performed poorly across both tests, while Model 3 showed improvement, with values for skewness and kurtosis near zero and three, respectively.

²⁵Stock prices and, particularly earnings, contributed to the lack of normality. The latter is likely a result of Shiller’s data on monthly earnings being interpolated from a quarterly frequency. However, the trace statistic and other inference in the CVAR model is more robust to non-normality and excess kurtosis than, say, high orders of skewness. See Juselius (2006), Chapter 4.

cointegrating relationship is significantly equilibrium-correcting for Δs_t . These results are strongly indicative of the predictions of the CEH-based model, though not all signs are as hypothesized. Figure 2 illustrates the graphs of the cointegrating relationships, $\hat{\beta}'_1 x_t$, $\hat{\beta}'_2 x_t$, and $\hat{\beta}'_3 x_t$, respectively. In all three cases, the cointegrating vectors appear to describe strongly stationary relations.

The size of the adjustment coefficients $\hat{\alpha}_{x,t}$ shed light on the speed of correction back to the equilibrium relation. Note that stock prices overshoot equilibrium-correction in $\hat{\beta}'_1$ and $\hat{\beta}'_3$. The adjustments of largest magnitude come from the social-context and psychology variables, with $\hat{\alpha}_{c,t}$ ranging from 0.197 to 1.750 and $\hat{\alpha}_{s,t}$ ranging from 0.206 to 1.535, which supports the finding, reported in Table 6, of a unit vector in alpha for social context and psychology. Stock-price adjustments range in magnitude from 0.130 to 0.333. The small magnitude of the adjustment coefficient for earnings supports the earlier finding of a common stochastic trend. Taken together, these results suggest that psychology and social context, in particular, are doing the lion’s share of equilibrium adjustment within the system, with marginally large effects stemming from stock prices, as implied by IKE and CEH.

4 Conclusion

This paper addresses the empirical failures of the traditional REH-based present-value model for stock prices by advancing an alternative representation of rationality. The CEH-based present-value model recognizes that rationality inherently involves social context and psychology and that market participants’ knowledge is imperfect and contingent. The theoretical structure and solution to the CEH-based present-value model for stock prices is provided. The implications of this model are tested in the cointegrated VAR framework which allows data characterized by non-stationary processes to “speak freely” in terms of the equilibrium driving forces and those which are error-correcting.

The information set formalizes the influence of social context and psychology underpinning market participants’ decision making by utilizing a novel dataset developed by Mangee (2011) which is based on information contained in *Bloomberg News*’ end-of-the-day wraps for the stock market. The textual data leave open when and in what way such considerations may matter for stock price fluctuations. This data in conjunction with the CVAR methodology allow for a nested-models approach to help determine what impacts structural change and social context and psychology have on the present-value relations for stock prices.

We find that we cannot reject the null of no cointegration between stock prices, earnings, and interest rates when we ignore structural change and dismiss the influence of social context and psychology on market participants’ forecasts. This finding is consistent with the literature. Allowing for mean shifts in the stock price relations leads to a marginal rejection of the null hypothesis. It is only when we allow for structural change in the cointegrating space *and* account for the influence of social context and psychology that we find strong evidence of an equilibrium relationship in the data. Since rationality necessarily involves social context and psychology and participants’ knowledge is contingent this is what we would expect to find.

Furthermore, earnings and interest rates are shown to be the primary driving forces of the cointegrating relationships. Movements in psychology and social context are found to be

equilibrium-correcting in response to changes in other variables. The informational variables enter the identified cointegrating space with significant coefficients and largely possess the hypothesized signs based on IKE and CEH. Overall, these findings suggest that the empirical struggles encountered by the traditional REH-based present-value model for stock prices is not the result of market participants' irrationality. Rather, the failures found in the literature stem from the inability of REH to capture rationality that is based on psychology and social context in a world where knowledge is contingent and grows.

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Table 1: Social Context Considerations

Political instability	Administration comments
IMF actions/comments	Trade agreements
Labor strike	Firm layoffs
Firm malpractice	Weakness in credit markets
Natural disaster	Analysts comments
Firm IPO	Fiscal policy
Fed chairman comments	SEC regulations
FDA announcement	Outbreak of disease
Terrorism	Firm bankruptcies
Firm M&A	Firm stock splits
Firm CEO actions/comments	Firm bailout
Firm stress tests	Airplane crash
Introduction of Euro	World Bank actions/comments
Central Bank reports	Armed conflicts
Political elections	ROW

Notes: See Mängee (2011) for details. IPO refers to initial public offering; FDA is Food and Drug Administration; ROW refers to any social context consideration pertaining to the rest of the world; M&A refers to mergers and acquisitions; IMF is International Monetary Fund.

Table 2: Psychological Considerations

Optimism	Concern
Pessimism	Euphoria
Confidence	Crowd psychology
Sentiment	Exuberance
Greed	Worry
Fear	Panic

Notes: See Mängee (2011) for a detailed description of these psychological considerations.

Table 3: Trace Tests for Cointegration

Model 1: $x'_t = [p, e, i]$					
$p - r$	r	Eig. Value	Trace	Frac95	P-Value
3	0	0.107	31.491	35.070	0.119
2	1	0.042	11.380	20.164	0.514
1	2	0.021	3.808	9.142	0.453

Model 2: $x'_t = [p, e, i]$ and ΦD_t					
$p - r$	r	Eig. Value	Trace	Frac95	P-Value
3	0	0.161	55.958	52.737	0.025
2	1	0.086	24.843	32.281	0.266
1	2	0.049	8.924	14.991	0.354

Model 3: $x'_t = [p, e, i, s, c]$ and ΦD_t					
$p - r$	r	Eig. Value	Trace	Frac95	P-Value
5	0	0.414	209.583	98.927	0.000
4	1	0.284	115.083	72.950	0.000
3	2	0.158	55.944	50.652	0.014
2	3	0.085	25.579	31.083	0.204
1	4	0.054	9.880	16.126	0.323

Table 4: Misspecification Tests

	Multivariate Tests				
	Autocorrelation			ARCH	
	LM(1)	LM(2)	Normality	LM(1)	LM(2)
Model 1: $x'_t = [p, e, i]$	13.275 (0.151)	12.853 (0.169)	232.618 (0.000)	82.203 (0.000)	106.423 (0.005)
Model 2: $x'_t = [p, e, i]$ and ΦD_t	9.503 (0.392)	10.512 (0.311)	124.259 (0.000)	60.883 (0.006)	96.102 (0.030)
Model 3: $x'_t = [p, e, i, s, c]$ and ΦD_t	30.145 (0.219)	30.334 (0.212)	135.900 (0.000)	200.682 (0.877)	427.945 (0.766)

Table 5: Common Stochastic Trends

	Δp	Δe	Δi	Δs	Δc
$\alpha'_{\perp 1}$	-0.209 (2.215)	1.000 (.NA)	0.000 (.NA)	0.017 (1.405)	0.008 (0.769)
$\alpha'_{\perp 2}$	-0.011 (-1.089)	0.000 (.NA)	1.000 (.NA)	-0.001 (-0.831)	-0.001 (-0.831)

Table 6: Test for Unit Vector in Alpha

r	DGF	5% CV	p_t	e_t	i_t	s_t	c_t
1	4	9.488	42.979 (0.000)	71.909 (0.000)	81.894 (0.000)	37.825 (0.000)	13.893 (0.008)
2	3	7.815	26.927 (0.000)	36.664 (0.000)	46.769 (0.000)	2.419 (0.490)	1.374 (0.712)
3	2	5.991	6.262 (0.044)	7.946 (0.019)	18.062 (0.000)	2.039 (0.361)	0.976 (0.614)
4	1	3.841	0.479 (0.489)	0.081 (0.775)	5.802 (0.016)	0.321 (0.571)	0.475 (0.491)

Figure 1: Time Series Graphs of x_t

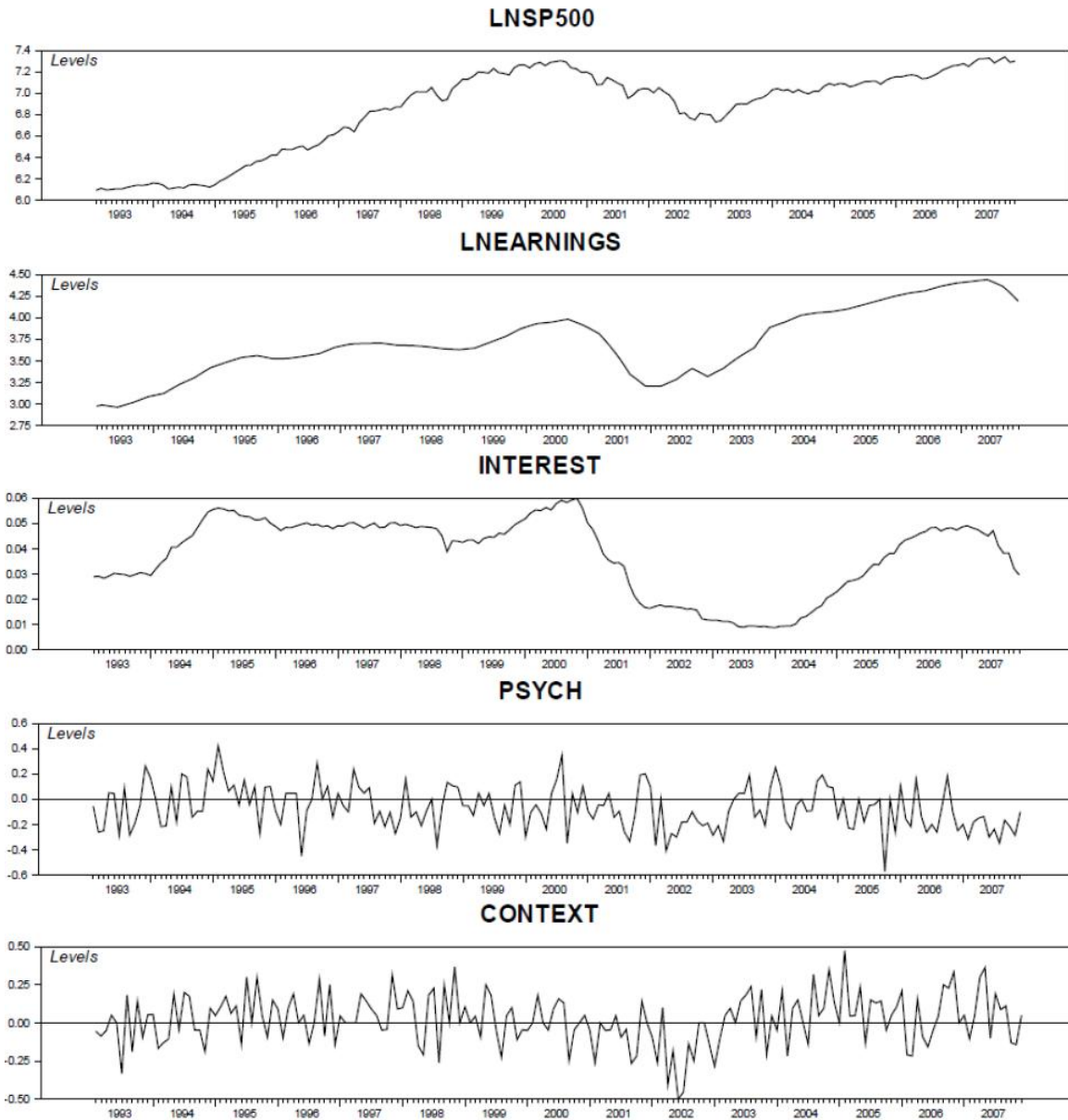
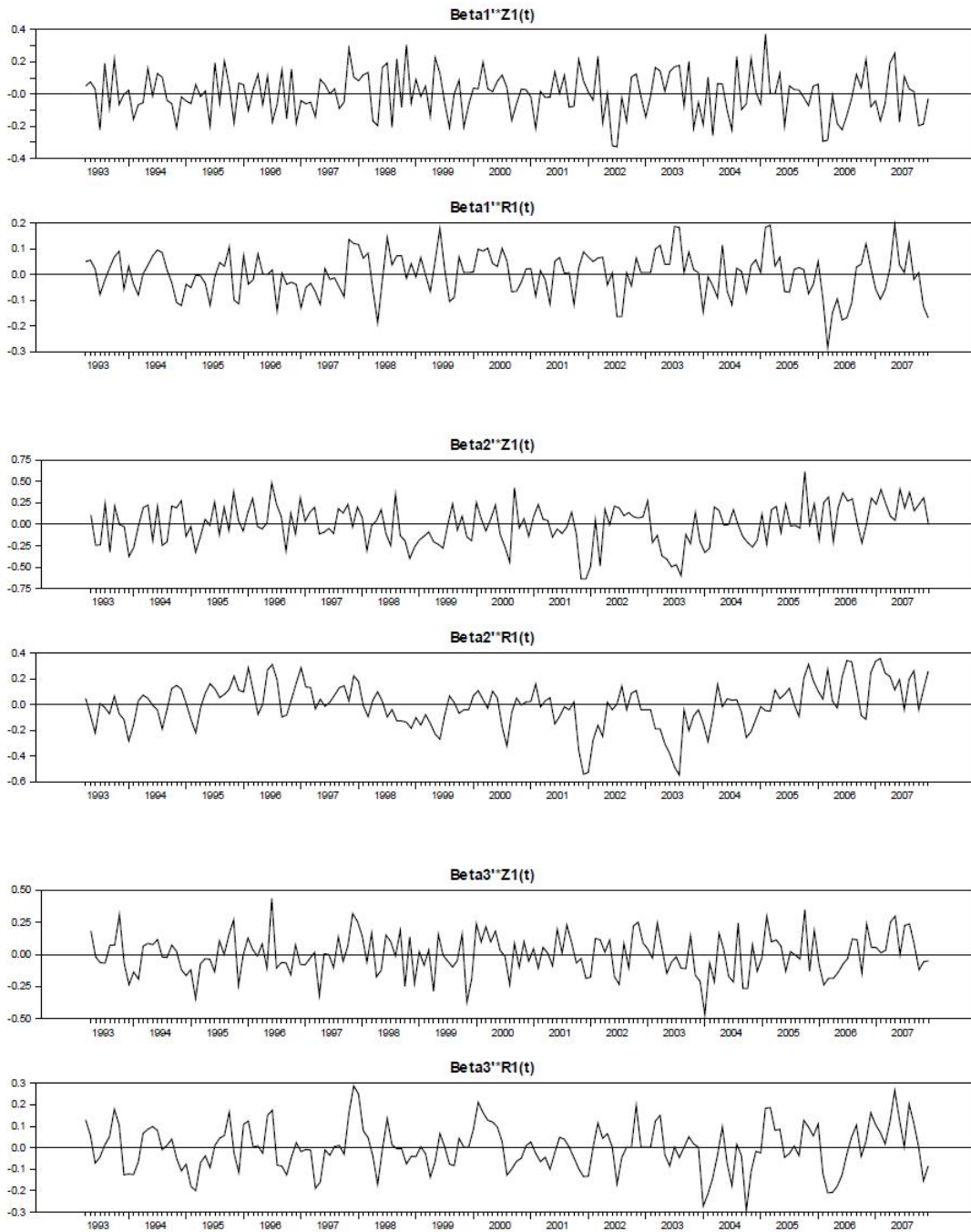


Figure 2: Cointegrating Relations $\hat{\beta}'_k x_t$



Notes: The upper panel illustrates $\hat{\beta}'_k x_t$ while the bottom panel illustrates $\hat{\beta}'_k R_t$ and corrects for short run effects.