USC Dornsife
Dana and David Dornsife College of Letters, Arts and Sciences

Institute for New Economic Thinking

USC Dornsife Institute for New Economic Thinking

Working Paper No. 15-18

Is There a Debt-threshold Effect on Output Growth?

Alexander Chudik, Kamiar Mohaddes, M. Hashem Pesaran and Mehdi Raissi

July 3, 2015

Is There a Debt-threshold Effect on Output Growth?*

Alexander Chudik[a], Kamiar Mohaddes[b], M. Hashem Pesaran[c], and Mehdi Raissi[d]

[a] Federal Reserve Bank of Dallas, USA

[b] Faculty of Economics and Girton College, University of Cambridge, UK

[c] USC Dornsife INET, Department of Economics, University of Southern California, USA and Trinity College, Cambridge, UK

[d] International Monetary Fund, Washington DC, USA

July 3, 2015

Abstract

This paper studies the long-run impact of public debt expansion on economic growth and investigates whether the debt-growth relation varies with the level of indebtedness. Our contribution is both theoretical and empirical. On the theoretical side, we develop tests for threshold effects in the context of dynamic heterogeneous panel data models with cross-sectionally dependent errors and illustrate, by means of Monte Carlo experiments, that they perform well in small samples. On the empirical side, using data on a sample of 40 countries (grouped into advanced and developing) over the 1965-2010 period, we find no evidence for a universally applicable threshold effect in the relationship between public debt and economic growth, once we account for the impact of global factors and their spillover effects. Regardless of the threshold, however, we find significant negative long-run effects of public debt build-up on output growth. Provided that public debt is on a downward trajectory, a country with a high level of debt can grow just as fast as its peers.

Keywords: Panel tests of threshold effects, long-run relationships, estimation and inference, large dynamic heterogeneous panels, cross-section dependence, debt, and inflation.

JEL Classifications: C23, E62, F34, H6.

*We are grateful to Luis Catão, Markus Eberhardt, Thomas Moutos, Kenneth Rogoff, Alessandro Rebucci, Ron Smith, Martin Weale, and Mark Wynne for helpful comments and suggestions. The views expressed in this paper are those of the authors and do not necessarily represent those of the Federal Reserve Bank of Dallas, the Federal Reserve System, the International Monetary Fund or IMF policy. Hashem Pesaran acknowledges financial support under ESRC Grant ES/I031626/1.

1 Introduction

The debt-growth nexus has received renewed interest among academics and policy makers alike in the aftermath of the recent global financial crisis and the subsequent euro area sovereign debt crisis. This paper investigates whether there exists a tipping point, for public indebtedness, beyond which economic growth drops off significantly; and more generally, whether a build-up of public debt slows down the economy in the long run. The conventional view is that having higher public debt-to-GDP can stimulate aggregate demand and output in the short run but crowds out private capital spending and reduces output in the long run. In addition, there are possible non-linear effects in the debt-growth relationship, where the build-up of debt can harm economic growth, especially when the level of debt exceeds a certain threshold, as estimated, for example, by Reinhart and Rogoff (2010) to be around 90% of GDP using a panel of advanced economies. However, such results are obtained under strong homogeneity assumptions across countries, and without adequate attention to dynamics, feedback effects from GDP growth to debt, and most importantly, error cross-sectional dependencies that exist across countries, due to global factors (including world commodity prices and the stance of global financial cycle) and/or spillover effects from one country to another which tend to magnify at times of financial crises.

Cross-country experience shows that some economies have run into debt difficulties and experienced subdued growth at relatively low debt levels, while others have been able to sustain high levels of indebtedness for prolonged periods and grow strongly without experiencing debt distress. This suggests that the effects of public debt on growth varies across countries, depending on country-specific factors and institutions such as the degree of their financial deepening, their track records in meeting past debt obligations, and the nature of their political system. It is therefore important that we take account of cross-country heterogeneity. Dynamics should also be modelled properly, otherwise the estimates of the long-run effects might be inconsistent. Last but not least, it is now widely agreed that conditioning on observed variables specific to countries alone need not ensure error cross-section independence that underlies much of the panel data literature. It is, therefore, also important that we allow for the possibility of cross-sectional error dependencies, which could arise due to omitted common effects, possibly correlated with the regressors. Neglecting such dependencies can lead to biased estimates and spurious inference. Our estimation strategy, outlined in Section 3, takes into account all these three features (dynamics, heterogeneity and cross-sectional dependence), in contrast with the earlier literature on debt-growth nexus.

In this paper we make both theoretical and empirical contributions to the cross-country analysis of the debt-growth relationship. We develop tests of threshold effects in dynamic panel data models and, by means of Monte Carlo experiments, illustrate that such tests perform well in the case of panels with small sizes typically encountered in the literature. In the empirical application, we specify a heterogeneous dynamic panel-threshold model and provide a formal statistical analysis of debt-threshold effects on output growth, in a relatively large panel of 40 countries, divided into advanced and developing economies, over the period 1965–2010. We study whether there is a common threshold for government debt ratios above which long-term growth rates drop off significantly, especially if the country is on an upward debt trajectory.[1] We do not find a universally applicable threshold effect in the relationship between debt and growth, for the full sample, when we account for error cross-sectional dependencies. Since global factors (including interest rates in the U.S., cross-country capital flows, global business cycles, and world commodity prices) play an important role in precipitating sovereign debt crises with long-lasting adverse effects on economic growth,[2] neglecting the resulting error cross-sectional dependencies can lead to spurious inference and false detection of threshold effects. Nonetheless, we find a statistically significant threshold effect in the case of countries with rising debt-to-GDP ratios beyond 50-60 percent, stressing the importance of debt trajectory. Provided that debt is on a downward path, a country with a high level of debt can grow just as fast as its peers. We find similar results, "no-simple-debt threshold", for the 19 advanced economies and 21 developing countries in our sample, as well as weak evidence of a debt trajectory effect in the case of advanced economies.

Another contribution of this paper is in estimating the long-run effects of public debt build-up on economic growth, regardless of whether there exists a threshold effect from debt-to-GDP ratio on output growth. It is shown that the estimates of long-run effects of debt accumulation on GDP growth are robust to feedbacks from growth to debt. Since in the case of some developing economies with relatively underdeveloped government bond markets, deficit financing is often carried out through money creation followed by high levels of inflation, we further investigate the robustness of our analysis by considering the simultaneous effects of inflation and debt on output growth. Like excessively high levels of debt, elevated inflation, when persistent, can also be detrimental for growth. By considering both inflation and debt we allow the regression analysis to accommodate both types of economies in the panel. Our results show that there are significant and robust negative long-run effects of debt ramp-up on economic growth, regardless of whether inflation is included in the various dynamic specifications examined. By comparison, the evidence of a negative effect of

[1] Due to the intrinsic cross-country heterogeneities, the debt thresholds are most-likely country specific and estimation of a universal threshold based on pooling of observations across countries might not be informative to policy makers interested in a particular economy. Relaxing the homogeneity assumption, whilst possible in a number of dimensions (as seen below), is difficult when it comes to the estimation of country-specific thresholds, because due to the non-linearity of the relationships involved, identification and estimation of country-specific thresholds require much larger time series data than are currently available. We therefore follow an intermediate approach where we test for the threshold effects not only for the full sample of countries but also for the sub-groups of countries (advanced economies and developing countries), assuming homogenous thresholds within each sub-group.

[2] For example, favorable terms of trade trends and benign external conditions typically lead to a borrowing ramp-up and pro-cyclical fiscal policy. When commodity prices drop or capital flows reverse, borrowing collapses and defaults occur followed by large negative growth effects.

inflation on growth is less strong, although it is statistically significant in the case of most specifications considered. In other words, if the debt level keeps rising persistently, then it will have negative effects on growth in the long run. On the other hand, if the debt-to-GDP ratio rises temporarily (for instance to help smooth out business cycle fluctuations), then there are no long-run negative effects on output growth. The key in debt financing is the reassurance, backed by commitment and action, that the increase in government debt is temporary and will not be a permanent departure from the prevailing norms.

The remainder of the paper is organized as follows. Section 2 formalizes the approach taken in Reinhart and Rogoff (2010) and reviews the literature. Section 3 presents our panel threshold model and develops panel tests of threshold effects for different model specifications. This section also provides small sample evidence on the performance of panel threshold tests. Section 4 presents the empirical findings on debt-threshold effects and the long-run impact of debt accumulation and inflation on economic growth. Some concluding remarks are provided in Section 5.

2 Reinhart and Rogoff's analysis of debt-threshold effects on output growth

The empirical literature on the relationship between debt and growth has, until recently, focused on the role of *external debt* in *developing countries*, with only a few studies providing evidence on developed economies.[3] A well-known influential example is Reinhart and Rogoff (2010), hereafter RR, who argue for a non-linear relationship, characterized by a threshold effect, between public debt and growth in a cross-country panel. It is useful to formalize the approach taken by these authors in order to outline the implicit assumptions behind its findings. RR bin annual GDP growths in a panel of 44 economies into four categories, depending on whether the debt is below 30% of GDP, between 30 to 60% of GDP, between 60 to 90% of GDP, or above 90% of GDP. Averages and medians of observations on annual GDP growth in each of the four categories are then reported. RR's main result is that the median growth rate for countries with public debt over 90% of GDP is around one percentage point per annum lower than median growth of countries with debt-to-GDP ratio below 90%. In terms of mean growth rates, this difference turns out to be much higher and amounts to around 4 percentage points per annum (Reinhart and Rogoff (2010), p. 575).

RR do not provide a formal statistical framework, but their approach can be characterized

[3]The predictions of the theoretical literature on the long-run effects of public debt on output growth are ambiguous, predicting a negative as well as a positive effect under certain conditions. Even if we rely on theoretical models that predict a negative relationship between output growth and debt, we still need to estimate the magnitude of such effects empirically. For an overview of the theoretical literature, see Chudik et al. (2013).

in the context of the following multi-threshold panel data model

$$\Delta y_{it} = \sum_{j=1}^{M} a_j I \left[\ln(\tau_{j-1}) < d_{it} \le \ln(\tau_j)\right] + e_{it}, \tag{1}$$

for $i = 1, 2, ..., N$, and $t = 1, 2, ..., T$, where Δy_{it} denotes the first difference of the logarithm of real GDP in country i during year t, d_{it} is the (natural) logarithm of debt-to-GDP ratio, M denotes the number of groups considered, τ_j for $j = 0, 1, ..., M$ are the threshold levels, $I(\mathcal{A})$ is an indicator variable that takes the value of unity if event $\mathcal{A}$ occurs and zero otherwise, with the end conditions, $I[d_{it} \le \ln(\tau_0)] = 0$, and $I[d_{it} \le \ln(\tau_M)] = 1$. In particular, RR set $M = 4$, $\tau_0 = -\infty$, $\tau_1 = 30\%$, $\tau_2 = 60\%$, $\tau_3 = 90\%$ and $\tau_4 = \infty$, thereby treating the threshold levels as given. RR's panel is unbalanced, but for expositional convenience we assume that the panel in (1) is balanced. It is easy to see that the indicator variables in (1) are orthogonal (since the four groups are mutually exclusive) and therefore the least squares (pooled) estimates of a_j for $j = 1, 2, ..., M$, in (1) are given by averages of Δy_{it} in the corresponding four groups, namely

$$\widehat{a}_j = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} \Delta y_{it} I \left[\ln(\tau_{j-1}) < d_{it} \le \ln(\tau_j)\right]}{\sum_{i=1}^{N} \sum_{t=1}^{T} I \left[\ln(\tau_{j-1}) < d_{it} \le \ln(\tau_j)\right]}, \text{ for } j = 1, 2, ...M.$$

As explained above, the main finding of RR is that $\widehat{a}_4$ (the average growth in the group with debt exceeding 90% of GDP) is several percentage points lower than other estimated means, $\widehat{a}_j$, for $j = 1, 2, 3$, which they find to be similar in magnitude.

Model (1) features multiple thresholds, which is more difficult to analyze than a single-threshold model. The hypothesis of interest (not formalized by RR) is that the average growth declines once the debt-to-GDP ratio exceeds a certain threshold. It is therefore more convenient to formalize this hypothesis in the context of the following parsimonious *single-threshold* model (assuming $M = 2$),

$$\Delta y_{it} = a_1 I \left[d_{it} \le \ln(\tau)\right] + a_2 I \left[d_{it} > \ln(\tau)\right] + e_{it}, \tag{2}$$

which can be written equivalently as

$$\Delta y_{it} = \alpha + \varphi I \left[d_{it} > \ln(\tau)\right] + e_{it}, \tag{3}$$

where $\alpha = a_1$ and $\varphi = a_2 - a_1$. There is a clear correspondence between the pooled estimates of (2) and those of (3). Pooled estimates of (3) can be motivated in a straightforward and intuitive manner by noting that $\hat{\alpha} = \hat{a}_1$ is the average output growth rate when the debt does not exceed the threshold ($d_{it} \le \ln(\tau)$), and $\hat{\varphi} = \hat{a}_2 - \hat{a}_1$ is the difference between the average output growth rate when the debt exceeds the threshold ($d_{it} > \ln(\tau)$) and the average output growth rate when the debt does not exceed the threshold ($d_{it} \le \ln(\tau)$). The hypothesis that the mean output growth rate declines once the debt threshold is exceeded corresponds to $\varphi < 0$ and φ measures the extent to which exceeding the threshold, τ, adversely affects the growth prospects. The null hypothesis of no threshold effect on output growth can then be investigated by testing the null hypothesis that $H_0 : \varphi = 0$ against the one-sided alternative that $H_1 : \varphi < 0$.

The analysis of RR has generated a considerable degree of debate in the literature. See, for example, Woo and Kumar (2015), Checherita-Westphal and Rother (2012), Eberhardt and Presbitero (2015), and Reinhart et al. (2012), who discuss the choice of debt brackets used, changes in country coverage, data frequency, econometric specification, and reverse causality going from output to debt.[4] These studies address a number of important modelling issues not considered by RR, but they nevertheless either employ panel data models that impose slope homogeneity and/or do not adequately allow for cross-sectional dependence across individual country errors. It is further implicitly assumed that different countries converge to their equilibrium at the same rate, and there are no spillover effects of debt overhang from one country to another. These assumptions do not seem plausible, given the diverse historical and institutional differences that exist across countries, and the increasing degree of interdependence of the economies in the global economy.

We shall build on (3) by allowing for endogeneity of debt and growth, fixed effects, dynamics (homogeneous and heterogeneous), as well as cross-sectional error dependence. We treat the threshold, τ, as an unknown parameter, and in developing a test of $H_0 : \varphi = 0$, we rigorously deal with the non-standard testing problem that arises, since τ is unidentified under the null hypothesis of no threshold effect. A satisfactory resolution of the testing problem is important since estimates of φ are statistically meaningful only if H_0 is rejected.

3 A panel threshold output growth model

We begin our econometric analysis with the following extension of (3)

$$\begin{aligned} \Delta y_{it} &= \alpha_{i,y} + \varphi I\left[d_{it} > \ln(\tau)\right] + \delta \Delta y_{i,t-1} + \eta \Delta d_{i,t-1} + e_{it}, \qquad (4) \\ \text{for } i &= 1, 2, ..., N, \text{ and } t = 1, 2, ..., T, \end{aligned}$$

and combine it with an equation for d_{it} (log of debt-to-GDP ratio)

$$\Delta d_{it} = \alpha_{i,d} + \rho d_{i,t-1} + \varkappa \Delta d_{i,t-1} + \psi \Delta y_{i,t-1} + \varepsilon_{it}, \qquad (5)$$

where we allow for feedbacks from lagged output growth to d_{it}. The idiosyncratic errors, e_{it} and ε_{it}, are assumed to be serially uncorrelated with zero means and heteroskedastic error variances. Both specifications include fixed effects, $\alpha_{i,y}$ and $\alpha_{i,d}$, but to simplify the exposi-

[4] See also Panizza and Presbitero (2013) for a survey and additional references to the literature.

tion we initially assume homogeneous slopes and cross-sectionally independent idiosyncratic errors. The debt equation allows for feedbacks from lagged output growth ($\psi \neq 0$), a unit root process for d_{it} when $\rho = 0$, and captures contemporaneous dependence between growth and debt via non-zero correlations between ε_{it} and e_{it}. To identify the threshold effects in the output growth equation we assume that no such threshold effects exist in the debt equation, (5). Nonetheless, we do not rule out the possibility of indirect threshold effects through the feedback variable, $\Delta y_{i,t-1}$.

It is important to note that even if τ was known, estimates of φ based on (4), would be subject to the simultaneity bias when ε_{it} is correlated with e_{it}, regardless of whether lagged variables are present in (4) and/or (5). The bias can be substantial, which we demonstrate by means of Monte Carlo experiments below. To deal with the simultaneity bias, we model the correlation between the two innovations and derive a reduced form equation which allows us to identify the threshold effect in the output equation, given that the threshold variable is excluded from the debt equation (our identification condition). To this end, assuming a linear dependence between the innovations, we have

$$e_{it} = \kappa_i \varepsilon_{it} + u_{it}, \tag{6}$$

where $u_{it} = e_{it} - E(e_{it} \,|\varepsilon_{it})$, and by construction u_{it} and ε_{it} are uncorrelated. The linearity of (6) is part of our identification assumption and is required if φ is to be estimated consistently. The coefficient κ_i measures the degree of simultaneity between output and debt innovations for country i. We allow κ_i to differ over i, considering the wide differences observed in debt-financing, and the degree to which automatic stabilizers offset fluctuations in economic activity across countries.

Substituting (6) in (4) and then substituting (5) for ε_{it}, we obtain the following "reduced form" panel threshold-ARDL specification for Δy_{it}:

$$\Delta y_{it} = c_i + \varphi I\left[d_{it} > \ln(\tau)\right] + \lambda_i \Delta y_{i,t-1} + \beta_{i0} \Delta d_{it} + \beta_{i1} \Delta d_{i,t-1} + \beta_{i2} d_{i,t-1} + u_{it}, \tag{7}$$

where $c_i = \alpha_{i,y} - \kappa_i \alpha_{i,d}$, $\lambda_i = \delta - \kappa_i \psi$, $\beta_{i0} = \kappa_i$, $\beta_{i1} = \eta - \kappa_i \varkappa$, and $\beta_{i2} = -\kappa_i \rho$. Since u_{it} is uncorrelated with ε_{it}, then conditional on $(\Delta y_{i,t-1}, \Delta d_{it}, \Delta d_{i,t-1}, d_{i,t-1})$, u_{it} and d_{it} will also be uncorrelated. From this and under our identification condition, it follows that u_{it} and $I\left[d_{it} > \ln(\tau)\right]$ will be uncorrelated and, hence, for a given value of τ, φ can be consistently estimated by filtered pooled least squares techniques applied to (7), after the fixed effects and the heterogeneous dynamics are filtered out. As we shall see below, the threshold coefficient, τ, can then be estimated by a grid search procedure. Since the focus of the analysis is on φ, assumed to be homogeneous, (7) can be estimated treating the other coefficients, c_i, λ_i, $\beta_{i0}, \beta_{i1}, \beta_{i2}$, as heterogeneous without having to impose the restrictions that exist across these coefficients due to the homogeneity of δ, ψ, η, ρ, and $\varkappa$, assumed under (4) and (5). Not imposing the cross-parameter restrictions in (7), when justified by the underlying slope homogeneity assumption, can lead to inefficient estimates and does not affect the consistency property of the filtered pooled estimators of φ and τ. In any case, the assumption that δ, ψ, η, ρ, and $\varkappa$ are homogenous across countries seems quite restrictive and imposing it could lead to inconsistent estimates of φ and τ.

Therefore, in what follows we base our estimation on (7), which deals with simultaneity bias, and allows for slope heterogeneity in the underlying output growth and debt equations. We shall also consider the possibility of cross-sectional error dependence below. Throughout we continue to assume that φ and τ are homogenous across countries, although we agree that in principle there are likely to be cross-country differences even for these parameters. To identify and estimate threshold parameters that differ across countries we need much longer time series data on individual countries and such data sets are available at most for one or two of the countries in our data set. Also, even if we did have long time series, there is no guarantee that for a given country-specific threshold, τ_i, there will be sufficient time variations in $I[d_{it} > \ln(\tau_i)]$ for a reliable estimation of a country-specific threshold effect coefficient, φ_i. In the empirical section below, we therefore follow an intermediate approach where we test for the threshold effects not only for the full sample of countries but also for the sub-groups of countries, assuming homogenous thresholds within each sub-group.

Since, in practice, any number of threshold variables could be considered, we allow for r threshold variables by replacing $\varphi I[d_{it} > \ln(\tau)]$ in (4) with $\boldsymbol{\varphi}'\mathbf{g}(d_{it}, \tau)$, where $\mathbf{g}(d_{it}, \tau) = [g_1(d_{it}, \tau), g_2(d_{it}, \tau), ..., g_r(d_{it}, \tau)]'$ is a vector of r threshold variables and $\boldsymbol{\varphi}$ is the $r \times 1$ vector of corresponding threshold coefficients. In this paper we focus on the following two threshold variables

$$g_1(d_{it}, \tau) = I[d_{it} > \ln(\tau)], \text{ and } g_2(d_{it}, \tau) = I[d_{it} > \ln(\tau)] \times \max(0, \Delta d_{it}), \tag{8}$$

where $g_1(d_{it}, \tau)$ is the standard threshold variable, and $g_2(d_{it}, \tau)$ is an interactive threshold variable, which takes a non-zero value only when d_{it} exceeds the threshold *and* the growth of debt-to-GDP is positive. Other combination of threshold effects can also be entertained.

Using (8), we have the following more general formulation of (7)

$$\Delta y_{it} = c_i + \boldsymbol{\varphi}'\mathbf{g}(d_{it}, \tau) + \lambda_i \Delta y_{i,t-1} + \beta_{i0}\Delta d_{it} + \beta_{i1}\Delta d_{i,t-1} + \beta_{i2} d_{i,t-1} + u_{it}, \tag{9}$$

which we refer to as panel threshold-ARDL model, and use below to develop panel tests of threshold effects.

3.1 Panel tests of threshold effects

Abstracting from the panel nature of (9), the problem of testing $\boldsymbol{\varphi} = \mathbf{0}$ is well known in the literature and results in non-standard tests since under $\boldsymbol{\varphi} = \mathbf{0}$, the threshold parameter τ disappears, and τ is only identified under the alternative hypothesis of $\boldsymbol{\varphi} \neq \mathbf{0}$. This testing problem was originally discussed by Davies (1977, 1987) and further developed in the econometrics literature by Andrews and Ploberger (1994) and Hansen (1996).

There exists only a few papers on the analysis of threshold effects in panel data models. Hansen (1999) considers the problem of estimation and testing of threshold effects in the case of static panels with fixed effects and homogeneous slopes, and deals with panels where the time dimension (T) is short and the cross section dimension (N) is large. He eliminates individual specific effects by de-meaning and as a result his approach cannot be extended to dynamic panels or panels with heterogeneous slopes. In a more recent paper, Seo and Shin (2014) allow for dynamics and threshold effects, but continue to assume slope homogeneity and use instruments to deal with endogeneity once the fixed effects are eliminated by first-differencing. Unlike these studies our focus is on panels with N and T large which allows us to deal with simultaneity, heterogeneous dynamics, and error cross-sectional dependence, whilst maintaining the homogeneity of the threshold parameters. Recall that we have already dealt with the endogeneity of the threshold variable, by considering a panel threshold-ARDL model where the threshold effects are identified by an exclusion restriction and the assumption that output growth and debt innovations are linearly related.

With the above considerations in mind, using vector notations, (9) for $t = 1, 2, ..., T$ can be written compactly as

$$\Delta \mathbf{y}_i = \mathbf{Q}_i \boldsymbol{\theta}_i + \boldsymbol{\varphi}' \mathbf{G}_i (\tau) + \mathbf{u}_i, \text{ for } i = 1, 2, ..., N, \tag{10}$$

where $\Delta \mathbf{y}_i$ is a $T \times 1$ vector of observations on Δy_{it}, $\mathbf{Q}_i$ is a $T \times h$ observation matrix of regressors $\mathbf{q}_{it} = (1, \Delta y_{i,t-1}, \Delta d_{it}, \Delta d_{i,t-1}, d_{i,t-1})'$, $h = 5$, and $\mathbf{G}_i (\tau)$ is a $T \times r$ matrix of observations on the threshold variables in $\mathbf{g} (d_{it}, \tau)$. The filtered pooled estimator of $\boldsymbol{\varphi}$ for a given value of τ is given by

$$\hat{\boldsymbol{\varphi}}(\tau) = \left[\sum_{i=1}^{N} \mathbf{G}_i' (\tau) \mathbf{M}_i \mathbf{G}_i (\tau) \right]^{-1} \sum_{i=1}^{N} \mathbf{G}_i' (\tau) \mathbf{M}_i \Delta \mathbf{y}_i,$$

where $\mathbf{M}_i = \mathbf{I}_T - \mathbf{Q}_i (\mathbf{Q}_i' \mathbf{Q}_i)^{-1} \mathbf{Q}_i$, and we refer to regressors in $\mathbf{Q}_i$ as the filtering variables. The set of filtering variables in $\mathbf{Q}_i$ depends on a particular specification of model (4) and (5), from which the empirical panel threshold-ARDL specification (9) is derived. The $SupF$ test statistic (see, for example, Andrews and Ploberger, 1994) for testing the null hypothesis $\boldsymbol{\varphi} = \mathbf{0}$ is given by

$$SupF = \sup_{\tau \in \mathcal{H}} \left[F_{NT}(\tau) \right], \tag{11}$$

where $\mathcal{H}$ represents the admissible set of values for τ and

$$F_{NT}(\tau) = \frac{(RSS_r - RSS_u) / r}{RSS_u / (n - s)},$$

in which RSS_u is the residual sum of squares of an unrestricted model (10), RSS_r is the residual sum of squares of the restricted model under the null $\boldsymbol{\varphi} = \mathbf{0}$, n is the number of available observations ($n = NT$), and s is the total number of estimated coefficients in the unrestricted model ($s = Nh + r$). Similarly, we define $AveF$ test statistics as

$$AveF = \frac{1}{\#\mathcal{H}} \sum_{\tau \in \mathcal{H}} F_{NT}(\tau), \tag{12}$$

where $\#\mathcal{H}$ denotes the number of elements of $\mathcal{H}$. The asymptotic distributions of the $SupF$ and $AveF$ test statistics are non-standard, but can be easily simulated. When $r = 1$ (e.g. when the threshold or the interactive threshold variables are considered separately), we use the square root of $F_{NT}(\tau)$ in (11) and (12) to obtain the $Sup\mathcal{T}$ and $Ave\mathcal{T}$ test statistics, respectively.

The above tests can be readily generalized to deal with possible correlation across the errors, u_{it}. Such error cross-sectional dependencies could arise due to spillover effects from cross-border trade or financial crises, or could be due to omitted common factors. There exits now considerable evidence suggesting that country macro-panels typically feature cross-sectionally correlated errors, and as we shall see, allowing for possible error cross-sectional dependencies is particularly important for our analysis where financial crises can have differential effects across countries, with the smaller and less developed economies being much more affected as compared to large economies.

We follow the literature and assume that u_{it}, the errors in (9), have the following multi-factor error structure

$$u_{it} = \boldsymbol{\gamma}_i' \mathbf{f}_t + v_{it}, \tag{13}$$

where $\mathbf{f}_t$ is the $m \times 1$ vector of unobserved common factors, which could themselves be serially correlated, $\boldsymbol{\gamma}_i$ is the $m \times 1$ vector of factor loadings, and v_{it}'s are the idiosyncratic errors which are uncorrelated with the factors, although they could be weakly cross-correlated. There are two ways of dealing with the presence of unobserved common factors in the literature. The factor space can be approximated by cross-sectional averages with either data-dependent or pre-determined weights. Examples of the former is a principal-components based approach by Song (2013), who extends the interactive effects estimator originally proposed by Bai (2009) to dynamic heterogeneous panels but does not provide any results on how to conduct inference on the means of the estimates of individual country-specific coefficients.[5] The latter approach is developed in the context of dynamic heterogeneous panels by Chudik and Pesaran (2015a). An advantage of using predetermined weights is that the properties of cross-sectional augmentation are easier to ascertain analytically and predetermined weights could lead to a better small sample performance. A recent overview of these methods and their relative merits is provided in Chudik and Pesaran (2015b). Following Chudik and

[5] Related is the quasi maximum likelihood estimator for dynamic panels by Moon and Weidner (2014), but this estimator has been developed only for homogeneous panels.

Pesaran (2015a), unobserved common factors can be dealt with in a straightforward manner by augmenting $\mathbf{Q}_i$ with the set of cross-section averages of output growth and debt variables, and their lags.

We document below that the small sample performance of the panel threshold tests proposed above are satisfactory in panels with or without unobserved common factors once $\mathbf{Q}_i$ is appropriately augmented by cross-section averages. We also show that the tests could be misleading when unobserved common factors are present and $\mathbf{Q}_i$ is not augmented with cross-section averages. In particular, we show that tests that do not account for the possibility of unobserved common factors could lead to the erroneous conclusion that threshold effects are present.

3.2 Small sample evidence on the performance of panel threshold tests

We now present evidence on the small sample performance of $SupF$ and $AveF$ tests defined in (11)–(12) (when $r > 1$), as well as the corresponding $Sup\mathcal{T}$ and $Ave\mathcal{T}$ tests statistics (when $r = 1$) and their extension to panels with multi-factor error structures defined by (13). We also illustrate the magnitude of the bias and size distortions in estimating φ and τ based on (4), that does not take account of the endogeneity of the threshold variable, and serves as the benchmark.

3.2.1 Monte Carlo design without common factors

Since the Sup and Ave tests are robust to heterogeneity of the slope coefficients in (4)–(5), we generate Δy_{it} as

$$\Delta y_{it} = \alpha_{i,y} + \varphi_1 g_1 \left(d_{it}, \tau\right) + \varphi_2 g_2 \left(d_{it}, \tau\right) + \delta_i \Delta y_{i,t-1} + \eta_i \Delta d_{i,t-1} + e_{it}, \tag{14}$$

where $g_1 \left(d_{it}, \tau\right)$ and $g_2 \left(d_{it}, \tau\right)$ are defined in (8), and the true value of τ is set to 0.8. We consider a heterogeneous version of (5) and omit $\Delta d_{i,t-1}$ for simplicity,

$$\Delta d_{it} = \alpha_{i,d} + \rho_i d_{i,t-1} + \psi_i \Delta y_{i,t-1} + \varepsilon_{it}, \tag{15}$$

where $e_{it} \sim IIDN\left(0, \sigma_{ei}^2\right)$. Let $\boldsymbol{\zeta}_{it} = \left(\Delta y_{it}, d_{it}\right)'$ and note that (14)–(15) can be equivalently written as a threshold VAR model,

$$\boldsymbol{\zeta}_{it} = \boldsymbol{\alpha}_i + \mathbf{A}_{i1} \boldsymbol{\zeta}_{i,t-1} + \mathbf{A}_{i2} \boldsymbol{\zeta}_{i,t-2} + \boldsymbol{\Phi} \mathbf{g}_{it}\left(\tau\right) + \mathbf{v}_{it}, \tag{16}$$

where $\mathbf{g}_{it}(\tau) = [g_1(d_{it},\tau), g_2(d_{it},\tau)]'$, $\boldsymbol{\alpha}_i = (\alpha_{i,y}, \alpha_{i,d})'$,

$$\mathbf{A}_{i1} = \begin{pmatrix} \delta_i & \eta_i \\ \psi_i & \rho_i + 1 \end{pmatrix}, \mathbf{A}_{i2} = \begin{pmatrix} 0 & -\eta_i \\ 0 & 0 \end{pmatrix}, \boldsymbol{\Phi} = \begin{pmatrix} \varphi_1 & \varphi_2 \\ 0 & 0 \end{pmatrix}, \text{ and } \mathbf{v}_{it} = \begin{pmatrix} e_{it} \\ \varepsilon_{it} \end{pmatrix}.$$

The dynamic processes Δy_{it} and d_{it} are generated based on (16) with 100 burn-in replications, and with zero starting values. In the absence of threshold effects (i.e. when $\boldsymbol{\Phi} = \mathbf{0}$), $\{\boldsymbol{\zeta}_{it}\}$ is stationary if $\rho(\boldsymbol{\Psi}_i) < 1$, for all i, where $\rho(\boldsymbol{\Psi}_i)$ denotes the spectral radius of $\boldsymbol{\Psi}_i$, and

$$\boldsymbol{\Psi}_i = \begin{pmatrix} \mathbf{A}_{i1} & \mathbf{A}_{i2} \\ \mathbf{I}_2 & \mathbf{0}_{2\times 2} \end{pmatrix}.$$

Moreover, in the absence of the threshold effects and assuming that $\{\boldsymbol{\zeta}_{it}$ for $i = 1, 2, ..., N\}$ have started in a distant past, then $E(\boldsymbol{\zeta}_{it}) = (\mathbf{I}_2 - \mathbf{A}_{i1}L - \mathbf{A}_{i2}L^2)^{-1}\boldsymbol{\alpha}_i$. We generate heterogeneous intercepts (fixed effects) as

$$\boldsymbol{\alpha}_i = (\mathbf{I}_2 - \mathbf{A}_{i1} - \mathbf{A}_{i2})\boldsymbol{\mu}_i, \ \boldsymbol{\mu}_i = \boldsymbol{\vartheta}_i + \xi \begin{pmatrix} 0.1 \\ 1 \end{pmatrix} \bar{\varepsilon}_{i\cdot},$$

$\boldsymbol{\vartheta}_i = (\vartheta_{i1}, \vartheta_{i2})'$, and $\bar{\varepsilon}_{i\cdot} = T^{-1}\sum_{t=1}^{T}\varepsilon_{it}$, which allows for some correlation between the individual effects and innovations of the debt equation. Finally, $\mathbf{v}_{it} = (e_{it}, \varepsilon_{it})'$ are generated as $\mathbf{v}_{it} \sim IIDN(\mathbf{0}, \boldsymbol{\Sigma}_v)$,

$$\boldsymbol{\Sigma}_v = E(\mathbf{v}_{it}\mathbf{v}_{it}') = \begin{pmatrix} \sigma_{ei}^2 & \mathfrak{r}_i\sigma_{ei}\sigma_{\varepsilon i} \\ \mathfrak{r}_i\sigma_{ei}\sigma_{\varepsilon i} & \sigma_{\varepsilon i}^2 \end{pmatrix},$$

which enables us to investigate the consequences of endogeneity of the threshold variables on the panel tests of the threshold effect.

We consider the following parameter configurations:

- **DGP1** (Baseline experiments without lags) $\rho_i = -1$, $\delta_i = 0$, $\psi_i = 0$, $\eta_i = 0$, and $\xi = 0$. We set $\mathfrak{r}_i = 0.5$, $\vartheta_{i1} = 0.03$, $\sigma_{\varepsilon i} = 1$ and generate $\sigma_{ei} \sim IIDU(0.01, 0.03)$ and $\vartheta_{i2} = IIDU(-0.9, -0.2)$. We set $\varphi_2 = 0$ and consider different options for $\varphi_1 \in \{-0.01, -0.009, .., 0, 0.001, ..., 0.01\}$ to study the size ($\varphi_1 = 0$) and the power ($\varphi_1 \neq 0$) of the $Sup\mathcal{T}$ and $Ave\mathcal{T}$ tests.

- **DGP2** (Experiments with lagged dependent variables in both equations) $\psi_i = 0$, $\eta_i = 0$, $\xi = 0$, $\delta_i \sim IIDU(0.2, 0.9)$ and $\rho_i \sim IIDU(-0.18, -0.02)$. We set $\mathfrak{r}_i = 0.5$, $\vartheta_{i1} \sim IIDU(0.01, 0.05)$, and generate $\sigma_{i,\varepsilon} = \sqrt{1-\rho_i^2}\kappa_{i,\varepsilon}$, $\kappa_{i,\varepsilon} \sim IIDU(0.8, 1.2)$, $\sigma_{i,e} \sim \sqrt{1-\delta_i^2}\kappa_{i,e}$, $\kappa_{i,e} \sim IIDU(0.01, 0.03)$ and $\vartheta_{i2} = IIDU(-0.9, -0.2)$. As in the previous DGP, we set $\varphi_2 = 0$ and consider different options for $\varphi_1 \in \{-0.01, -0.009, .., 0, 0.001, ..., 0.01\}$.

- **DGP3** (Experiments featuring lagged dependent variables and feedback effects) $\delta_i \sim IIDU(0.2, 0.9)$, $\eta_i \sim IIDU(0, 0.02)$, $\rho_i \sim IIDU(-0.18, -0.02)$ and $\psi_i \sim IIDU(0, 1)$.

The remaining parameters are generated as $\mathfrak{r}_i \sim IIDU(0,1)$, $\xi = 1$, $\vartheta_{i1} \sim IIDU(0.01, 0.05)$, $\sigma_{i,\varepsilon} = \sqrt{1-\rho_i^2}\kappa_{i,\varepsilon}$, $\kappa_{i,\varepsilon} \sim IIDU(0.8, 1.2)$, $\sigma_{i,e} \sim \sqrt{1-\delta_i^2}\kappa_{i,e}$, $\kappa_{i,e} \sim IIDU(0.01, 0.03)$ and $\vartheta_{i2} = IIDU(-0.9, -0.2)$. As in the previous DGPs, we set $\varphi_2 = 0$ consider different options for $\varphi \in \{-0.01, -0.009, .., 0, 0.001, ..., 0.01\}$.

- **DGP4** (Same as DGP3 but with an interactive indicator) δ_i, η_i, ρ_i, ψ_i, $\mathfrak{r}_i$, ξ, $\boldsymbol{\vartheta}_i$, $\sigma_{i,\varepsilon}$, and $\sigma_{i,e}$ are generated in the same way as in DGP3. We set $\varphi_1 = 0$ and consider different options for $\varphi_2 \in \{-0.01, -0.009, .., 0, 0.001, ..., 0.01\}$.

3.2.2 Monte Carlo design with common factors

We extend the set of Monte Carlo designs in the previous subsection by generating data using factor-augmented versions of (14)–(15), namely

$$\Delta y_{it} = \alpha_{i,y} + \varphi_1 g_1(d_{it}, \tau) + \varphi_2 g_2(d_{it}, \tau) + \delta_i \Delta y_{i,t-1} + \eta_i \Delta d_{i,t-1} + \boldsymbol{\gamma}_{i,y}' \mathbf{f}_t + e_{it}, \tag{17}$$

and

$$\Delta d_{it} = \alpha_{i,d} + \rho_i d_{i,t-1} + \psi_i \Delta y_{i,t-1} + \boldsymbol{\gamma}_{i,x}' \mathbf{f}_t + \varepsilon_{it},$$

where $\mathbf{f}_t = (f_{1t}, f_{2t})'$ is a 2-dimensional vector of unobserved common factors. Intercepts ($\alpha_{i,y}$ and $\alpha_{i,d}$) and errors (e_{it} and ε_{it}) are generated in the same way as in the experiments without factors. The factors and their loadings are generated as

- **DGP5** (Factor-augmented version of DGP3) $\mathbf{f}_t \sim IID(\mathbf{0}, \mathbf{I}_2)$, $\boldsymbol{\gamma}_{i,y} = (\gamma_{i1,y}, 0)'$, $\boldsymbol{\gamma}_{i,d} = (0, \gamma_{i2,d})'$, $\gamma_{i1,y} \sim IIDN(\gamma_y, \sigma_{\gamma y}^2)$, $\gamma_{i2,d} \sim IIDN(\gamma_d, \sigma_{\gamma d}^2)$, $\gamma_y = 0.01$, $\sigma_{\gamma y} = 0.01$, $\gamma_d = 0.1$ and $\sigma_{\gamma d} = 0.1$. The remaining parameters are generated in the same way as in DGP3.

- **DGP6** (No threshold effects in the output equation and factors subject to threshold effects) Unobserved common factors are generated as

$$f_{1t} = \varphi_f I\left[\bar{d}_t > \ln(\tau)\right] + v_{f1t}, \tag{18}$$

$$f_{2t} = v_{f2t}, \tag{19}$$

where $\bar{d}_t = \sum_{i=1}^N d_{it}$ and $\mathbf{v}_{ft} = (v_{f1t}, v_{f2t})' \sim IIDN(\mathbf{0}, \mathbf{I}_2)$. Factor loadings are generated as $\boldsymbol{\gamma}_{i,y} = (\gamma_{i1,y}, 0)'$, $\boldsymbol{\gamma}_{i,d} = (0, \gamma_{i2,d})'$, $\gamma_{i1,y} = \gamma_{i2,d}/100 + \vartheta_{i,y}$, $\vartheta_{i,y} \sim IIDN(0.01, 0.01^2)$, and $\gamma_d = \sigma_{\gamma d} = 1$. We set $\varphi_1 = \varphi_2 = 0$ and $\varphi_f = -1$. The remaining parameters are generated in the same way as in DGP1.

Remark 1 *Under DGP5 the incidence of the threshold effect is country-specific with no threshold effect in the unobserved common factors, whilst under DGP6 any observed threshold effect is due to the common factor. Using these two DGPs we will be able to investigate the effectiveness of the cross-sectional augmentation techniques to deal with the presence of* *common factors (irrespective of whether the common factors are subject to threshold effects or not), and illustrate the consequence of ignoring cross-sectional error dependence when there are in fact common factors subject to threshold effects.*

3.2.3 Monte Carlo findings

First we present findings for the baseline experiments (DGP1), where $\boldsymbol{\zeta}_{it} = (\Delta y_{it}, d_{it})'$ is given by the simple model without lags

$$y_{it} = \alpha_{i,y} + \varphi_1 g_1(d_{it}, \tau) + e_{it}, \tag{20}$$

$$d_{it} = \alpha_{i,d} + \varepsilon_{it}. \tag{21}$$

Table 1 reports Bias ($\times 100$) and RMSE ($\times 100$) of estimating $\varphi_1 = -0.01$, and $\tau = 0.8$. We consider the pooled and fixed effects (FE) estimators based on (20), and the filtered pooled estimator described in Subsection 3.1 with the vector of filtering variables $\mathbf{q}_{it} = (1, d_{it})'$, to take account of the contemporaneous dependence in the innovations of (20) and (21). It can be seen from Table 1 that, for the baseline DGP1, both pooled and FE estimators are substantially biased due to the non-zero correlation of output growth and debt innovations. By contrast, the filtered pooled estimator exhibits little bias and its RMSE declines with N and T as expected. The power functions of $Sup\mathcal{T}$ and $Ave\mathcal{T}$ tests and standard t-tests computed for three selected assumed values of τ (namely 0.2, 0.5 and 0.9) are shown in Figure 1 in the case of the experiments with $N = 40$ and $T = 46$ (this sample size pair is chosen since they approximately match the sample sizes encountered in the empirical application). The individual t-tests are included for comparison. The figure shows the rejection rates for $\varphi_1 \in \{-0.01, -0.009, .., 0\ (\text{size})\ , 0.001, ..., 0.01\}$. All six tests have the correct size (set at 5%), but it is clear that both $Sup\mathcal{T}$ and $Ave\mathcal{T}$ tests have much better power properties, unless the value of τ selected *a priori* in the construction of the standard t-tests is very close to the unknown true value. It is clear that $Sup\mathcal{T}$ and $Ave\mathcal{T}$ perform well without knowing the true value of τ, although there is little to choose between $Sup\mathcal{T}$ and $Ave\mathcal{T}$; both tests perform well.

Similar satisfactory results are obtained for the filtered pooled estimators of φ and τ, under DGP2 and DGP3, which allow for dynamics and feedback effects. The same is also true of $Sup\mathcal{T}$ and $Ave\mathcal{T}$ tests of φ in the case of these DGPs. For brevity, these results are reported in a supplement, which is available upon request.

Next, we investigate estimation and inference in the case of DGP4, which features a lagged dependent variable, feedback effects and an interactive threshold variable. In these experiments, we estimate $\boldsymbol{\varphi} = (\varphi_1, \varphi_2)' = (0, -0.01)'$ and conduct $SupF$ and $AveF$ tests defined in (11)–(12). Table 2 gives the results for the Bias ($\times 100$) and RMSE ($\times 100$) of the filtered pooled estimators using $\mathbf{q}_{it} = (1, \Delta y_{i,t-1}, \Delta d_{it}, \Delta d_{it-1}, d_{i,t-1})'$ as the filtering variables. These results clearly show that the proposed estimation method works well even if N and T are relatively small (around 40). The biases of estimating φ_1 and φ_2 are small and the associated RMSEs fall steadily with N and T. The threshold parameter, τ, is even more precisely estimated. For example, in the case of experiments with $N = T = 40$, the bias of estimating $\tau = 0.80$ is -0.0006, and falls to -0.0001 when $N = T = 100$, with RMSE declining quite rapidly with N and T. The tests of the threshold effects perform very well as well. Figure 2 shows the power functions of testing $\varphi_1 = 0$ in the case of the experiments with $N = 40$ and $T = 46$, using $SupF$ and $AveF$ testing procedures.

Results on small sample performance of $SupF$ and $AveF$ tests in the case of DGP5 with unobserved common factors, are quite similar to the findings in the case of DGP3 and are provided in the supplement. The shape of the power function is the same as in Figure 1 and the size distortion is relatively small, although slightly larger as compared compared with the empirical sizes obtained under DGP3. This could be due to the small T time series bias and a larger number of coefficients that are estimated under DGP5 (due to cross-section augmentation). The findings for the Bias ($\times 100$) and RMSE ($\times 100$) of estimating $\varphi_1 = -0.01$, and $\tau = 0.8$ in the case of DGP5 are also reported in the supplement. The results are quite similar to those obtained for DGP3. The bias is small for all sample sizes considered and the RMSE improves with an increase in N and/or T.

More interesting are the results for the panel threshold tests in the case of DGP6, which does not feature threshold effects in the output equation (namely $\varphi_1 = \varphi_2 = 0$ in equation (17)), but the unobserved common factor, f_{1t}, is subject to a threshold effect, as specified by (18). We conduct the $SupF$ and $AveF$ tests without augmentation by cross-sectional averages (reported on the left panel of Table 3), as well as with augmentation by cross-section averages (reported on the right panel of Table 3). Tests without cross-section (CS) augmentation show large size distortions, 63% to 93%, depending on the sample size, suggesting that erroneous evidence for threshold effects could be obtained if we do not account for the unobserved common factors. On the other hand, $SupF$ and $AveF$ tests with CS augmentation perform as expected, showing only slight size distortions with empirical sizes in the range of 9% to 12% for $T = 46$, and 6% to 8% for $T = 100$. Bias and RMSE for φ_1 reported at the lower part of the table show evidence of inconsistency of the estimates without CS augmentation (the bias is substantial and increases with increases in N and T), whereas the bias is virtually zero when the filtered pooled estimation procedure is carried out with CS augmentation. It is clear that CS augmentation is critical for avoiding spurious inference in the case of panels with error cross-sectional dependence.

4 Empirical findings

In this section, we provide a formal statistical analysis of debt-threshold effects on output growth, using a relatively large panel of 40 countries over the period 1965–2010. We allow for country-specific heterogeneity in dynamics, error variances, and cross-country correlations, but assume homogeneous threshold parameters. To shed some light on possible heterogeneity of the threshold effects across countries, we also report separate results for the 19 advanced and 21 developing economies. In the case of CS augmented estimates, cross-section averages are computed using all available observations across all 40 countries in the sample. Furthermore, we examine the long-term effects of public debt build-up on economic growth using both ARDL and DL specifications discussed in Chudik et al. (2015), as well as their cross-sectionally augmented versions. Finally, we examine the robustness of our main findings by including inflation in our empirical analysis. This is important, because in the case of some developing economies with limited access to international debt markets, deficit financing through domestic money creation, and hence inflation, might be a more important factor in constraining growth than government debt.

We use public debt at the general government level for as many countries as possible, but given the lack of general public debt data for many countries, central government debt data is used as an alternative. The construction of data and the underlying sources are described in the Data Appendix. Since our analysis allows for slope heterogeneity across countries, we need a sufficient number of time periods to estimate country-specific coefficients. To this end, we include only countries in our sample for which we have at least 30 consecutive annual observations on debt and GDP. Subject to this requirement we ended up with the 40 countries listed in Table A.1. These countries cover most regions in the world and include advanced, emerging and developing economies. To account for error cross-sectional dependence, we need to form cross-section averages based on a sufficient number of units, and hence set the minimum cross-section dimension to 10. Overall, we ended up with an unbalanced panel covering 1965-2010, with $T_{\min} = 30$, and $N_{\min} = 10$ across all countries and time periods.[6]

4.1 Tests of the debt-threshold effects

Reinhart and Rogoff (2010) and Checherita-Westphal and Rother (2012) argue for the presence of threshold effects in the relationship between debt-to-GDP and economic growth. However, as already noted, RR's analysis is informal and involves comparisons of average growth rate differentials across economies classified by their average debt-to-GDP ratios. They find that these differentials peak when debt-to-GDP ratio is around 90-100%. Krugman (1988) and Ghosh et al. (2013) also consider possible threshold effects in the relationship between external debt and output growth, which is known as debt overhang. However, these results are based on strong homogeneity restrictions, zero feedback effects from GDP growth to debt, no dynamics, and independence of cross-country errors terms.

To explore the importance of heterogeneities, simultaneous determination of debt and growth, and dynamics, we begin with the following baseline autoregressive distributed lag

[6] See Section 7 in Chudik and Pesaran (2015b) for further details on the application of the Common Correlated Effects (CCE) estimators to unbalanced panels.

(ARDL) specification, which extends (9) to p lags,

$$\Delta y_{it} = c_i + \boldsymbol{\varphi}'\mathbf{g}\left(d_{it}, \tau\right) + \sum_{\ell=1}^{p} \lambda_i \Delta y_{i,t-\ell} + \sum_{\ell=0}^{p} \beta_{i\ell} \Delta d_{i,t-\ell} + v_{it}, \tag{22}$$

and, following Chudik et al. (2015), we also consider the alternative approach of estimating the long-run effects using the distributed lag (DL) counterpart of (22), given by

$$\Delta y_{it} = c_i + \boldsymbol{\theta}'\mathbf{g}\left(d_{it}, \tau\right) + \phi_i \Delta d_{it} + \sum_{\ell=0}^{p} \alpha_{i\ell} \Delta^2 d_{i,t-\ell} + v_{it}, \tag{23}$$

where $\mathbf{g}\left(d_{it}, \tau\right)$ consists of up to two threshold variables: $g_1\left(d_{it}, \tau\right) = I\left[d_{it} > \ln(\tau)\right]$ and/or $g_2\left(d_{it}, \tau\right) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$. The threshold variable $g_1\left(d_{it}, \tau\right)$ takes the value of 1 if debt-to-GDP ratio is above the given threshold value of τ and zero otherwise. The interactive threshold term, $g_2\left(d_{it}, \tau\right)$, is non-zero only if $\Delta d_{it} > 0$, and $d_{it} > \ln(\tau)$. As before, y_{it} is the log of real GDP and d_{it} is the log of debt-to-GDP. In addition to assuming a common threshold, τ, specifications (22) and (23) also assume that the coefficients of the "threshold variables", $\boldsymbol{\varphi}$ and $\boldsymbol{\theta}$, are the same across all countries whose debt-to-GDP ratio is above the common threshold τ. We test for the threshold effects not only in the full sample of 40 countries, but also for the two sub-samples of advanced and developing countries, assuming homogenous thresholds within each group, but allowing for the threshold parameters to vary across the country groupings.

As explained in Chudik et al. (2015), sufficiently long lags are necessary for the consistency of the ARDL estimates, whereas specifying longer lags than necessary can lead to estimates with poor small sample properties. The DL method, on the other hand, is more generally applicable and only requires that a truncation lag order is selected. We use the same lag order, p, for all variables/countries but consider different values of p in the range of 1 to 3 for the ARDL approach and 0 to 3 for the DL method, to investigate the sensitivity of the results to the choice of the lag order. Given that we are working with growth rates which are only moderately persistent, a maximum lag order of 3 should be sufficient to fully account for the short-run dynamics. Furthermore, using the same lag order across all variables and countries help reduce the possible adverse effects of data mining that could accompany the use of country and variable specific lag order selection procedures such as the Akaike or Schwarz criteria. Note that our primary focus here is on the long-run estimates rather than the specific dynamics that might be relevant to a particular country.

The test outcomes of debt-threshold effects are summarized in Table 4 for all countries, in Table 5 for advanced economies, and in Table 6 for developing economies. Each table contains three panels, giving the *Sup* and *Ave* test statistics for the joint and separate tests of threshold effects. Panel (a) reports the *SupF* and *AveF* test statistics for the joint statistical significance of both threshold variables $[g_1\left(d_{it}, \tau\right)$ and $g_2\left(d_{it}, \tau\right)]$; panel (b) gives the test results for the significance of the simple threshold variable, $g_1(d_{it}, \tau)$; and panel (c) provides the test results for the significance of the interactive threshold variable, $g_2(d_{it}, \tau)$. The left panel of each table gives the test results based on the ARDL and DL specifications, (22) and (23), whilst the right panels give the results for the ARDL and DL specifications augmented with cross-section averages, denoted by CS-ARDL and CS-DL, respectively.

The test results differ markedly depending on whether the ARDL and DL specifications are augmented with cross-section averages, and to a lesser degree on the choice of the country grouping under consideration. For the full sample and when the panel regressions are not augmented with cross-section averages, the tests results are statistically significant in all cases, irrespective of the choice of the lag order and the estimation procedure (ARDL or DL). Similar results follow when we consider the two country groupings separately, although the strength of the results depends on the choice of the estimation method, with the DL procedure strongly rejecting the null of no threshold effects (in line with the full sample results), whilst the tests based on the ARDL regressions are mixed (see Tables 5 and 6).

Overall, there appears to be some support for debt-threshold effects using ARDL and DL specifications, with the estimates of the thresholds being $60-80$ percent for the full sample, 80 percent for the advanced economies, and between $30-60$ percent for the developing countries, see panel (b) of Tables 4 to 6. Interestingly, the threshold effects for advanced economies at 90 percent and for developing countries at 60 percent calculated in Reinhart and Rogoff (2010) and elsewhere in the literature are close to those reported in Tables 5 and 6. Note also that, consistent with the literature, the debt-to-GDP thresholds appear to be significantly lower for developing economies as opposed to those of advanced countries.

Although specifications (22) and (23) deal with heterogeneity, endogeneity, and dynamics, they do not allow for error cross-sectional dependence. We need to be cautious when interpreting these results as both panel ARDL and DL methodologies assume that the errors in the debt-growth relationships are cross-sectionally independent, which is likely to be problematic as there are a number of factors, such as trade and financial integration, external-debt financing of budget deficits, the stance of global financial cycle, and exposures to common shocks (i.e. oil price disturbances), that could invalidate such an assumption. These global factors are mostly unobserved and can simultaneously affect both domestic growth and public debt, and as was illustrated by Monte Carlo experiments above, can lead to biased estimates if the unobserved common factors are indeed correlated with the regressors.

To investigate the extent of error cross-sectional dependence, in Tables 4–6 we report the cross-section dependence (CD) test of Pesaran (2004, 2015), which is based on the average of pair-wise correlations of the residuals from the underlying ARDL and DL regressions.[7] For

[7] Theoretical properties of the CD test have been established in the case of strictly exogenous regressors and pure autoregressive models. The properties of the CD test for dynamic panels that include lagged dependent variables and other (weakly or strictly exogenous) regressors have not yet been investigated. However, the Monte Carlo findings reported in Chudik et al. (2015) suggest that the CD test continues to

all lag orders, we observe that these residuals display a significant degree of cross-sectional dependence. Under the null of weak error cross-sectional dependence, the CD statistics are asymptotically distributed as $N(0,1)$, and are highly statistically significant, particularly for advanced economies and all the 40 countries together.

Given the strong evidence of error cross-sectional dependence, and as shown in Section 3, the panel threshold tests based on ARDL and DL regressions that do not allow for error cross-sectional dependence can yield incorrect inference regarding the presence of threshold effects. To address this problem, we employ the CS-ARDL and CS-DL approaches, based on Chudik and Pesaran (2015a) and Chudik et al. (2015), which augment the ARDL and DL regressions with cross-sectional averages of the regressors, the dependent variable and their lags. Specifically, the cross-sectionally augmented ARDL (CS-ARDL) specification is given by

$$\Delta y_{it} = c_i + \boldsymbol{\varphi}'\mathbf{g}\left(d_{it},\tau\right) + \sum_{\ell=1}^{p}\lambda_i\Delta y_{i,t-\ell} + \sum_{\ell=0}^{p}\beta_{i\ell}\Delta d_{i,t-\ell} + \sum_{\ell=0}^{p}\boldsymbol{\omega}'_{i\ell,h}\overline{\mathbf{h}}_{t-\ell} + \boldsymbol{\omega}'_{i,g}\overline{\mathbf{g}}_t\left(\tau\right) + u_{it}, \quad (24)$$

where $\overline{\mathbf{h}}_t = \left(\overline{\Delta y}_t, \overline{\Delta d}_t\right)'$, $\overline{\Delta y}_t$ and $\overline{\Delta d}_t$ are defined as averages of output growth and debt-to-GDP growth across countries, and other variables are defined as before. The cross-sectionally augmented DL (CS-DL) specification is defined by

$$\Delta y_{it} = c_i + \boldsymbol{\theta}'\mathbf{g}\left(d_{it},\tau\right) + \phi_i\Delta d_{it} + \sum_{\ell=0}^{p}\alpha_{i\ell}\Delta^2 d_{i,t-\ell} + \omega_{i,y}\overline{\Delta y}_t + \sum_{\ell=0}^{p}\omega_{i\ell,d}\overline{\Delta d}_{t-\ell} + \boldsymbol{\omega}'_{i,g}\overline{\mathbf{g}}_t\left(\tau\right) + u_{it}. \quad (25)$$

Compared to the CS-ARDL approach, the CS-DL method has better small sample performance for moderate values of T, which is often the case in applied work, see Chudik et al. (2015).[8] Furthermore, it is robust to a number of departures from the baseline specification, such as residual serial correlation, and possible breaks in the error processes.

The tests based on the CS-ARDL and CS-DL regressions are summarized on right panels of Tables 4 to 6. First, the CD test statistics for CS-ARDL and CS-DL models, confirm a substantial decline in the average pair-wise correlation of residuals after the cross-section augmentation of the ARDL and DL models. Second, considering the joint tests in panel (a) we note that while there is some support for debt-threshold effects for all countries (Table 4), this is somewhat weaker for the advanced economies (Table 5) as the *Sup* and *Ave* tests are not always statistically significant, and in fact the joint tests are not statistically significant (irrespective of the lag order or the estimation method) in the case of developing economies (Table 6). Third, and in sharp contrast to the estimates based on (22) and (23), the test results based on CS-ARDL and CS-DL estimates in Panel (b) of Tables 4 to 6,

be valid even when the panel data model contains lagged dependent variable and other regressors.

[8] The sampling uncertainty in the CS-ARDL model could be large when the time dimension is moderate and the performance of the estimators also depends on a correct specification of the lag orders of the underlying ARDL specifications.

do not reject the null of no simple debt-threshold effects, once we allow for cross-sectional error dependence. However, for the full sample of 40 countries, the interactive threshold variable, $g_2(d_{it}, \tau) = I[d_{it} > \ln(\tau)] \times \max(0, \Delta d_{it})$, continues to be statistically significant with τ estimated in the range $40 - 60$ percent. See the CS-ARDL and CS-DL estimates in panel (c) of Table 4. These results suggest that *debt trajectory* is probably more important for growth than the *level* of debt itself. Support for a debt trajectory effect is also found for the advanced economies group in panel (c) of Table 5, although the threshold estimates are now rather poorly estimated and fall in a wide range, 10% (for $1 \leq p \leq 3$) to 100% (for $p = 0$), depending on the lag order selected.

In this regard, the evidence for the developing economies, summarized on the right-hand side of panel (c) of Table 6, is even weaker. Once the regressions are augmented with cross-sectional averages, the null hypothesis that there is no interactive threshold effects cannot be rejected. This could be due to the small number of countries in the group combined with a much greater degree of heterogeneity across developing economies, as compared to the advanced countries. The economies in this group are also less developed financially, which could be another contributory factor.

To summarize, the panel threshold tests based on the ARDL and DL specifications provide evidence for a threshold effect (in the range of $60 - 80$ percent) in the the relationship between public debt and growth, with this threshold being significantly smaller for developing economies (between $30 - 60$ percent) as opposed to those of advanced countries (80 percent). However, once we account for the possible effects of common unobserved factors and their spillovers, we are not able to find a universally applicable threshold effect. This fits nicely with the results in Section 3 showing that when unobserved common factors are present and the ARDL and DL regressions are not augmented with cross-section averages, statistical evidence of threshold effects might be spurious. It is important that the residuals from standard panel regressions are tested for cross-sectional error dependence and the robustness of the threshold tests to augmentation with cross-section averages investigated.

Finally, we thought it important to check the robustness of our results to the inclusion of inflation in our analysis. We have singled out inflation, since in many countries in the panel that do not have developed bond markets, government deficit is often financed through money creation with subsequent high inflation, and little change in debt-to-GDP levels. By considering both inflation and debt, we allow the regression analysis to accommodate both types of economies in the panel. The panel threshold tests for this extended set up are reported in Table 7, from which we see that, overall, the results echo those obtained without π_{it} as a regressor in Table 4: once we consider the CS-ARDL and CS-DL specifications there is no evidence for a debt-threshold effect, although we find that debt-trajectory is important especially when $\tau > 50\%$. We also did the same analysis for the two sub-groups, (i) 19 advanced economies and (ii) 21 developing economies, and found very similar results to those reported in Tables 5 and 6. For brevity, these results are not reported in the paper but are available in the supplement.

4.2 Estimates of long-run effects

The above analysis suggests that, once we account for the impact of global factors and their spillover effects, there is only a weak evidence for a universally applicable threshold effect in the relationship between public debt and economic growth, with the threshold variable being statistically significant only when it is interacted with a positive change in debt-to-GDP. However, our main object of interest is not only testing for the presence of threshold effects but ultimately the estimation of the long-run effects of a persistent increase in debt-to-GDP on output growth, regardless of whether there is a threshold effect. To investigate this, we first consider the long-run effects of debt accumulation on output growth using the ARDL and DL specifications in equations (22) and (23). In a series of papers, Pesaran and Smith (1995), Pesaran (1997), and Pesaran and Shin (1999) show that the traditional ARDL approach can be used for long-run analysis, and that the ARDL methodology is valid regardless of whether the regressors are exogenous, or endogenous, and irrespective of whether the underlying variables are $I(0)$ or $I(1)$. These features of the panel ARDL approach are appealing as reverse causality could be very important in our empirical application. While a high debt burden may have an adverse impact on economic growth, low GDP growth (by reducing tax revenues and increasing government expenditures on unemployment and welfare benefits) could also lead to high debt-to-GDP ratios. We are indeed interested in studying the relationship between public debt build-up and output growth after accounting for these possible feedback effects. We also utilize the DL approach for estimating the long-run relationships for its robustness. Both ARDL and DL specifications allow for a significant degree of cross-county heterogeneity and account for the fact that the effect of an increase in public debt and inflation on growth could vary across countries (particularly in the short run), depending on country-specific factors such as institutions, geographical location, or cultural heritage.

The least squares estimates obtained from the panel ARDL and DL specifications are reported in Table 8 for three cases: (i) full sample, (ii) advanced economies, and (iii) developing countries.[9] Panel (a) reports the estimation results for models with both threshold variables, $g_1(d_{it}, \tau)$ and $g_2(d_{it}, \tau)$, included. Panels (b) and (c) show the results when the threshold variables, $g_1(d_{it}, \tau)$ and $g_2(d_{it}, \tau)$, are included separately. Panel (d) reports the results without the threshold variables. Each panel gives the Mean Group (MG) estimates of the long-run effects of debt-to-GDP growth, Δd_{it}, on GDP growth. As shown in Pesaran and Smith (1995), the MG estimates are consistent under fairly general conditions so long as the errors are cross-sectionally independent. The results across all specifications suggest an inverse relationship between a change in debt-to-GDP and economic growth. Specifically,

[9] Individual country estimates are available on request, but it should be noted that they are likely to be individually unstable given the fact that the time dimension of the panel is relatively small.

Table 8 shows that the coefficients of debt-to-GDP growth, $\widehat{\phi}_{\Delta d}$, are negative and mostly statistically significant at the 1 percent level, with their values ranging from -0.04 to -0.11 across various groups, estimation techniques (ARDL and DL), and lag orders.

However, as noted above, we need to check the robustness of the long-run estimates to possible error cross-sectional dependence. Using the CD test statistics (reported in Table 4–6) we note that the error terms across countries in the ARDL and DL regressions exhibit a considerable degree of cross-sectional dependence that are highly statistically significant for all lag orders. As before, to overcome this problem, we re-estimated the long-run coefficients using the CS augmented versions of ARDL and DL. The estimation results are summarized in Table 8, where we provide the MG estimates for the four specifications, (a)–(d), discussed above. For all specifications, we note that $\widehat{\phi}_{\Delta d}$ is generally larger than in the ARDL and DL regressions, ranging between -0.03 and -0.15, and still statistically significant at the 1 percent level in most cases. In fact, out of the 168 coefficients reported in Table 8, only 9 are insignificant. Therefore, it appears that there are significant negative long-run effects of public debt build-up on growth, irrespective of whether threshold variables are included. These results suggest that if the debt-to-GDP ratio keeps growing, then it will have negative effects on economic growth in the long run. Provided that debt is on a downward path, a country with a high level of debt can grow just as fast as its peers.

Similar to the panel threshold tests, we conducted robustness checks by including inflation as an additional regressor in the different specifications. The estimation results are summarized in Table 9, where we provide the least squares estimates for the four different cases, (a)–(d), discussed above. Each panel gives the Mean Group (MG) estimates of the long-run effects of debt-to-GDP growth and inflation on GDP growth (denoted by $\widehat{\phi}_{\Delta d}$ and $\widehat{\phi}_{\pi}$). We note that the coefficients of $\widehat{\phi}_{\Delta d}$ is negative and statistically significant at the 1 percent level in all cases, for all four specifications (ARDL, DL, and their cross-sectionally augmented versions), and for all lag orders. Specifically, Table 9 shows that the coefficients of debt-to-GDP growth is in the range of -0.05 to -0.10 (across various panels) based on the DL and ARDL models, while $\widehat{\phi}_{\Delta d}$ is somewhat larger, ranging between -0.06 and -0.10, when considering the cross-sectionally augmented versions of DL and ARDL. Turning to the long-run effects of inflation on growth we notice that in the case of DL and ARDL estimations $\widehat{\phi}_{\pi}$ is between -0.04 and -0.08, while the CS-ARDL and CS-DL estimates of $\widehat{\phi}_{\pi}$ lie in the range of -0.08 and -0.20, being larger than those obtained from ARDL and DL regressions, as the latter does not take into account the possibility that the unobserved common factors are correlated with the regressors. Note that the CD test statistics in Table 9 confirm a substantial decline in the average pair-wise correlation of residuals after the cross-section augmentation of the ARDL and DL models. Furthermore, once we have appropriately augmented the regressions with the cross-sectional averages of the relevant variables we now have more evidence for negative growth effects of inflation in the long run as the CS-ARDL and CS-DL estimates are significant (at the 1% level) in most cases. Overall, the results suggest that, once we account for the impact of global factors and their spillover effects, like excessively high levels of debt, high levels of inflation, when persistent, can also be detrimental for growth.

One drawback of the CS-DL approach is that the estimated long-run effects are only consistent when the feedback effects from the lagged values of the dependent variable to the regressors are absent, although Chudik et al. (2015) argue that, even with this bias, the performance of CS-DL in terms of RMSE is much better than that of the CS-ARDL approach when T is moderate (which is the case in our empirical application). Having said that, it should be noted that no one estimator is perfect and each technique involves a trade-off. Estimators that effectively address a specific econometric problem may lead to a different type of bias. For instance, while the CS-DL estimator is capable of dealing with many modeling issues (cross sectional dependencies, robustness to different lag-orders, serial correlations in errors, and breaks in country-specific error processes), it leaves the feedback effects problem unresolved. To deal with different types of econometric issues, and to ensure more robust results, we conducted the debt-inflation-growth exercise based on two estimation methods (CS-ARDL and CS-DL). We note that the direction/sign of the long-run relationship between a change in debt and growth is always negative and statistically significant (across different specification and lag orders). This is also the case for the relationship between inflation and growth in most of the models estimated.

5 Concluding remarks

The effect of public debt accumulation on growth is central in the policy debate on the design of optimal fiscal policies that balance the short-run gains from fiscal expansion and possible adverse effects on growth in the long run. This topic has received renewed interest among economists and policy makers in the aftermath of the global financial crisis and the European sovereign debt crisis. This paper revisited the question of the long-run effect of debt accumulation on growth, and its dependence on indebtedness levels, in a dynamic heterogeneous and cross-sectionally correlated unbalanced panel of countries.

We first developed tests for threshold effects in the context of large dynamic heterogeneous panel data models with cross-sectionally dependent errors and, by means of Monte Carlo experiments, illustrated that they perform well in small samples. We then provide a formal statistical analysis of debt threshold effects on output growth by applying these tests to a panel of 40 countries, as well as to two sub-groups of advanced and developing economies, over the period 1965–2010. We were not able to find a universally applicable simple threshold effect in the relationship between public debt and growth once we accounted for the effects of global factors. However, we did find statistically significant threshold effects in the case of countries with rising debt-to-GDP ratios. These results suggest that the *debt trajectory* can have more important consequences for economic growth than the *level* of debt-to-GDP itself. Moreover, we showed that, regardless of debt thresholds, there is a significant negative long-run relationship between *rising* debt-to-GDP and economic growth. Our results imply that the Keynesian fiscal deficit spending to spur growth does not necessarily have negative long-run consequences for output growth, so long as it is coupled with credible fiscal policy plan backed by action that will reduce the debt burden back to sustainable levels.

Tables and Figures

Table 1: MC findings for Bias(x100) and RMSE(x100) of the estimation of φ_1 and τ in the baseline experiments without lags (DGP1)

	Pooled estimators				Fixed effects estimators				Filtered pooled estimators			
	Bias (x100)		RMSE (x100)		Bias (x100)		RMSE (x100)		Bias (x100)		RMSE (x100)	
(N,T)	**46**	**100**	**46**	**100**	**46**	**100**	**46**	**100**	**46**	**100**	**46**	**100**
	φ_1 (true value $= -0.01$)											
40	1.5137	1.5117	1.5190	1.5152	1.5410	1.5415	1.5464	1.5450	0.0172	0.0065	0.1416	0.0962
100	1.5095	1.5091	1.5117	1.5105	1.5399	1.5395	1.5421	1.5409	0.0054	0.0011	0.0909	0.0614
	τ (true value = 0.80)											
40	-74.36	-74.73	74.37	74.73	-74.52	-74.79	74.52	74.79	0.00	0.00	1.55	0.70
100	-74.77	-74.89	74.77	74.89	-74.80	-74.93	74.80	74.93	-0.02	0.00	0.61	0.25

Notes: Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it})'$ as the vector of filtering variables.

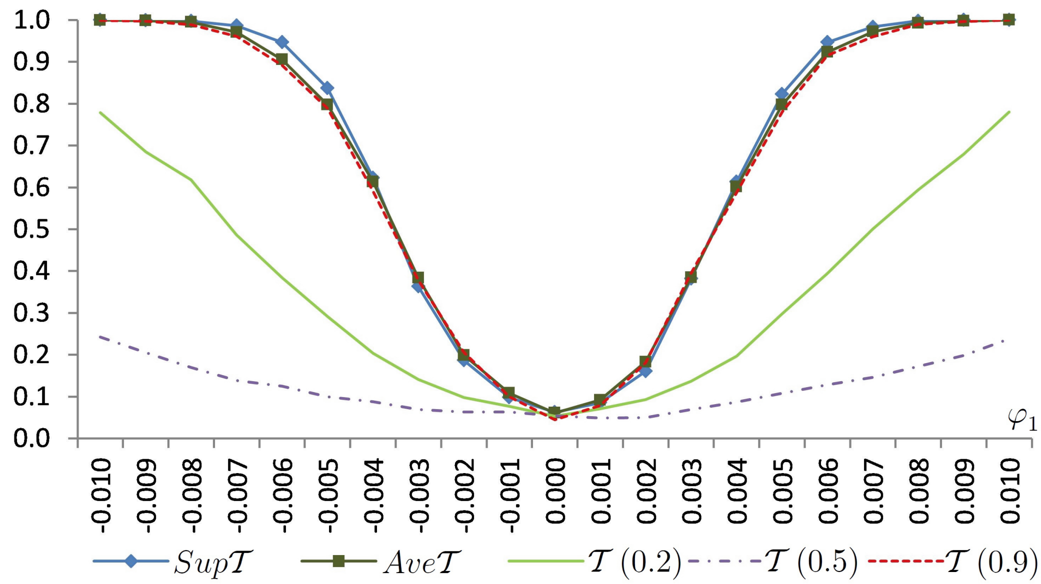
Table 2: MC findings for Bias(x100) and RMSE(x100) for the estimation of φ_1, φ_2, and τ in experiments with lagged dependent variable, feedback effects and two threshold indicators (DGP4)

	Filtered pooled estimators			
	Bias (x100)		RMSE (x100)	
(N,T)	**40**	**100**	**40**	**100**
	φ_1 (true value = 0.0)			
40	0.0227	0.0123	0.1489	0.0926
100	0.0190	0.0100	0.1029	0.0600
	φ_2 (true value $= -0.01$)			
40	-0.0070	-0.0025	0.1362	0.0864
100	-0.0070	-0.0023	0.0891	0.0550
	τ (true value = 0.8)			
40	-0.06	-0.03	1.66	0.65
100	-0.01	-0.01	0.59	0.27

Notes: Filtered pooled estimators are computed using the vector of filtering variables, $\mathbf{q}_{it} = (1, \Delta y_{i,t-1}, d_{i,t-1}, \Delta d_{it}, \Delta d_{it-1})'$.

Figure 1: Power functions of $Sup\mathcal{T}$ and $Ave\mathcal{T}$ tests for testing the null of $\varphi_1 = 0$ against the alternatives $\varphi_1 \in \{-0.01, -0.009, .., 0, 0.001, ..., 0.01\}$ in DGP1

$N = 40$, and $T = 46$,



Notes: $Sup\mathcal{T}$ and $Ave\mathcal{T}$ are *Sup* and *Ave*, *t*-tests of $\varphi_1 = 0$ in DGP1, with rejection frequencies computed at $\varphi_1 = -0.01, 0.009, ..., 0.0, 0.001, ..., 0.009, 0.01$. $\mathcal{T}(\tau)$ is the *t*-test of the threshold effect ($\varphi_1 = 0$) computed for three *a priori* selected values of τ, $\tau = 0.2, 0.5$ and 0.9.

Table 3: MC findings for the estimation of φ_1 and τ in DGP6 (experiments without threshold effects [$\varphi_1 = \varphi_2 = 0$] and with unobserved common factors subject to threshold effects)

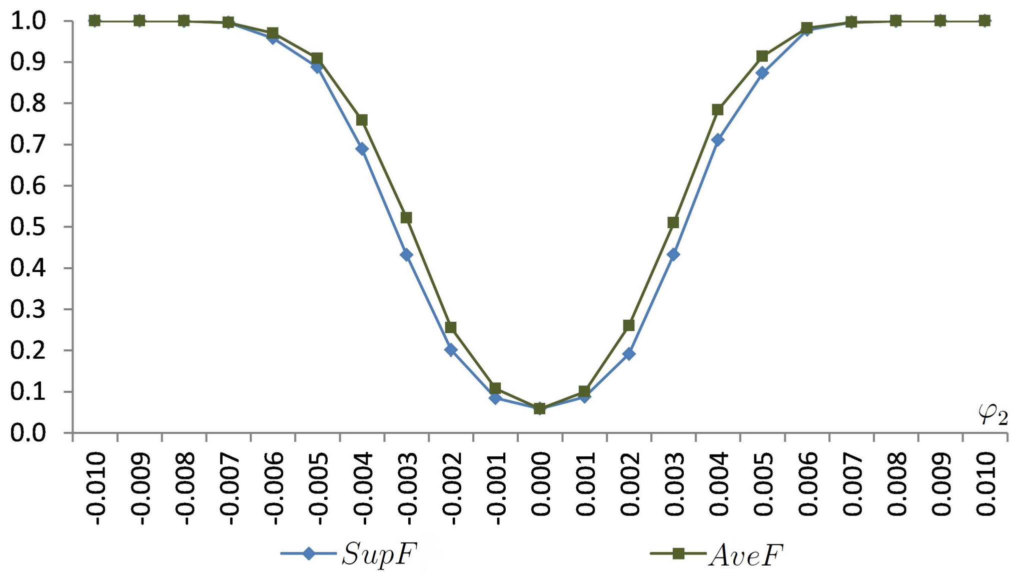
Rejection rates for $Sup\mathcal{T}$ and $Ave\mathcal{T}$ tests, and bias(x100) and RMSE(x100) for estimates of φ_1

	Without CS augmentation				**With CS augmentation**			
(N,T)	**40**	**100**	**40**	**100**	**40**	**100**	**40**	**100**
	Rejection rates of $Sup\mathcal{T}$ and $Sup\mathcal{T}$ tests							
	$Sup\mathcal{T}$		$Ave\mathcal{T}$		$Sup\mathcal{T}$		$Ave\mathcal{T}$	
40	63.45	79.60	65.55	81.35	12.10	7.55	9.90	7.65
100	83.25	92.05	83.45	92.90	9.95	7.65	9.20	6.60
	Bias(x100) and RMSE(x100) for estimates of φ_1							
	Bias (x100)		RMSE (x100)		Bias (x100)		RMSE (x100)	
40	0.2344	0.2211	1.1033	0.8628	0.0000	-0.0041	0.2258	0.1428
100	0.3034	0.3725	1.0245	0.8554	0.0008	0.0004	0.1375	0.0863

Notes: Filtered pooled estimators without cross-section (CS) augmentation are computed using $\mathbf{q}_{it} = (1, d_{it})'$ as the vector of filtering variables, and the filtered pooled estimators with CS augmentation are computed using the vector of filtering variables, $\mathbf{q}_{it} = \left(1, d_{it}, , \bar{\boldsymbol{\zeta}}_t', \bar{\boldsymbol{\zeta}}_{t-1}'\right)'$ where $\bar{\boldsymbol{\zeta}}_t$ is the arithmetic cross-sectional average of $\zeta_{it} = [d_{it}, \Delta y_{it}, g_{1it}(\tau)]'$.

Figure 2: Power functions for $SupF$ **and** $AveF$ **tests for testing the null of** $\varphi_1 = \varphi_2 = 0$ **against the alternatives** $\varphi_1 = 0$**,** $\varphi_2 \in \{-0.01, -0.009, ..., 0, 0.001, ..., 0.01\}$ **in the case of DGP4**

$N = 40$, and $T = 46$



Notes: $SupF$ and $AveF$ are Sup and Ave, F-tests of $\varphi_1 = \varphi_2 = 0$ in DGP4, with rejection frequencies computed at $\varphi_1 = 0$ and for $\varphi_2 = -0.01, 0.009, ..., 0.0, 0.001, ..., 0.009, 0.01$.

Table 4: Tests of debt-threshold effects for *all countries*, 1966-2010

	ARDL			DL				CS-ARDL			CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	(3,3,3)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$														
$\widehat{\tau}$	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.40	0.40	0.30	0.40	0.40	0.40	0.40
SupF	22.82‡	32.16‡	26.51‡	37.36‡	36.72‡	38.51‡	47.26‡	15.94†	12.68	12.64	18.79‡	18.18‡	16.87†	13.63
AveF	15.25‡	18.65‡	15.62‡	23.60‡	21.42‡	22.21‡	24.02‡	7.36‡	5.46†	5.80*	9.21‡	10.03‡	8.11‡	8.20‡
CD	17.95	15.41	15.44	21.54	17.32	13.96	13.51	-1.40	-0.88	0.22	-1.08	-1.19	-2.04	-0.98
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$														
$\widehat{\tau}$	0.80	0.60	0.60	0.80	0.80	0.60	0.60	0.40	0.30	0.30	0.40	0.40	0.40	0.40
$Sup\mathcal{T}$	3.24†	3.98‡	3.23*	5.22‡	5.19‡	5.24‡	6.14‡	3.15*	2.12	2.20	2.92*	2.67	2.16	1.49
$Ave\mathcal{T}$	2.24‡	2.57‡	2.04‡	3.90‡	3.67‡	3.75‡	4.07‡	1.14	0.93	0.91	1.16*	0.90	0.71	0.77
CD	18.57	15.68	15.66	22.52	18.73	14.25	13.97	-1.14	-0.75	-0.04	-0.85	-1.07	-1.90	-1.24
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$														
$\widehat{\tau}$	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.60	0.40	0.60	0.60	0.60	0.50
$Sup\mathcal{T}$	4.74‡	5.62‡	5.14‡	5.97‡	5.65‡	6.04‡	6.62‡	2.80	2.99	3.16	2.86	3.23†	3.33†	3.44†
$Ave\mathcal{T}$	3.79‡	4.12‡	3.79‡	4.54‡	4.23‡	4.34‡	4.5‡	1.96‡	1.85‡	1.88‡	2.34‡	2.5‡	2.34‡	2.48‡
CD	17.98	15.49	15.44	22.17	17.95	14.44	14.02	-1.34	-1.03	-0.03	-1.11	-1.30	-2.06	-1.60

Notes: The ARDL and DL specifications are given by (22) and (23) while the CS-ARDL and CS-DL specifications are given by (24) and (25). Panel (a) reports the *SupF* and *AveF* test statistics for the joint statistical significance of both threshold variables $[g_1(d_{it},\tau)$ and $g_2(d_{it},\tau)]$, while panel (b) and (c) reports the $Sup\mathcal{T}$ and $Ave\mathcal{T}$ test statistics for the statistical significance of the simple threshold variable $g_1(d_{it},\tau)$, and the interactive threshold variable, $g_2(d_{it},\tau)$, respectively. Statistical significance of the *Sup* and *Ave* test statistics is denoted by *, † and ‡, at 10%, 5% and 1% level, respectively. CD is the cross-section dependence test statistic of Pesaran (2004).

Table 5: Tests of debt-threshold effects for *advanced economies*, 1966-2010

	ARDL			DL				CS-ARDL			CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	(3,3,3)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it}(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$														
$\widehat{\tau}$	0.60	0.60	0.60	0.80	0.80	0.80	0.80	0.10	0.10	0.10	0.20	0.20	0.10	0.20
SupF	19.18‡	26.19‡	24.24‡	25.37‡	30.19‡	35.90‡	39.75‡	6.56	5.54	12.99	11.39*	9.71	9.08	15.45*
AveF	10.91‡	13.24‡	12.36‡	15.72‡	18.02‡	19.84‡	19.83‡	3.00	3.49	6.77†	4.76‡	4.24†	4.28*	6.87‡
CD	18.39	15.91	15.89	23.81	18.75	16.76	15.58	4.56	3.48	2.07	13.72	8.54	3.77	3.46
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$														
$\widehat{\tau}$	0.80	0.80	0.80	0.80	0.80	0.80	0.80	0.40	0.40	0.10	0.20	0.20	0.20	1.00
$Sup\mathcal{T}$	2.67	2.87	2.70	4.45‡	4.91‡	5.28‡	5.39‡	1.68	1.98	2.57	2.43	1.96	2.32	2.44
$Ave\mathcal{T}$	1.75‡	1.87‡	1.45‡	3.52‡	3.74‡	3.79‡	3.69‡	1.02	0.99	1.26	1.20*	0.98	1.02	1.24*
CD	18.58	16.61	16.48	24.50	19.66	17.42	16.71	6.78	5.92	3.57	13.22	9.23	6.73	5.21
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$														
$\widehat{\tau}$	0.60	0.60	0.60	0.80	0.80	0.60	0.80	0.10	0.10	0.10	1.00	0.10	0.10	0.10
$Sup\mathcal{T}$	4.31‡	5.08‡	4.82‡	4.75‡	5.03‡	5.92‡	6.09‡	2.44	2.83	3.21	3.51†	3.33†	3.53†	4.23‡
$Ave\mathcal{T}$	2.99‡	3.32‡	3.23‡	3.20‡	3.40‡	4.00‡	4.17‡	1.38†	1.7‡	2.23‡	1.68‡	1.89‡	1.91‡	2.45‡
CD	18.38	15.92	15.87	24.11	19.12	16.78	15.47	5.38	3.75	2.53	10.51	8.00	3.92	2.50

Notes: See the notes to Table 4.

Table 6: Tests of debt-threshold effects for *developing economies*, 1966-2010

	ARDL			DL				CS-ARDL			CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	(3,3,3)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$														
$\widehat{\tau}$	0.50	0.50	0.50	0.50	0.50	0.30	0.30	0.40	0.50	0.50	0.20	0.40	0.50	0.50
SupF	13.23†	15.66†	13.93*	17.25‡	14.16†	14.61†	17.96‡	9.15	2.38	2.86	9.37	7.76	6.40	4.07
AveF	7.94‡	9.33‡	7.27‡	11.56‡	8.89‡	8.24‡	9.82‡	2.47	0.93	1.11	2.86	2.94	2.10	1.95
CD	4.53	4.28	3.67	5.48	4.32	3.25	3.33	-2.24	-1.50	-1.03	-2.02	-2.30	-2.02	-1.59
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$														
$\widehat{\tau}$	0.50	0.60	0.50	0.60	0.50	0.30	0.30	0.40	0.50	0.30	0.50	0.40	0.40	0.40
SupT	2.52	3.10*	2.91	3.50‡	3.31†	3.44†	4.06‡	2.83	1.69	1.67	2.75	2.70	2.14	1.47
AveT	1.71‡	2.02‡	1.6‡	2.51‡	2.18‡	2.23‡	2.61‡	1.19	0.78	0.75	1.02	1.26*	0.72	0.63
CD	4.85	4.65	4.23	5.59	4.58	3.88	3.99	-2.25	-1.46	-0.93	-1.90	-2.21	-1.63	-1.14
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$														
$\widehat{\tau}$	0.50	0.50	0.50	0.60	0.60	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50
SupT	3.57†	3.87‡	3.67†	4.11‡	3.52†	3.43†	3.92‡	1.66	1.69	1.39	1.53	1.32	1.73	1.44
AveT	2.70‡	2.87‡	2.53‡	3.28‡	2.82‡	2.58‡	2.66‡	0.53	0.38	0.63	0.70	0.59	0.80	0.87
CD	4.67	4.46	3.73	5.84	4.72	3.61	3.69	-2.60	-1.42	-1.62	-1.89	-2.52	-1.91	-1.50

Notes: See the notes to Table 4.

Table 7: Tests of debt-threshold effects for *all countries* (robustness to the inclusion of inflation in the regressions), 1966-2010

	ARDL			DL				CS-ARDL		CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$													
$\widehat{\tau}$	0.50	0.60	0.60	0.50	0.60	0.60	0.60	0.40	0.40	0.40	0.40	0.40	0.40
SupF	23.57‡	25.45‡	25.36‡	21.21‡	21.71‡	16.77†	27.67‡	15.61*	15.78	16.14†	14.56*	18.58*	20.49*
AveF	13.84‡	12.49‡	11.15‡	13.2‡	12.35‡	9.61‡	12.84‡	6.78‡	6.67†	6.46‡	7.05‡	7.07‡	8.05‡
CD	20.36	16.33	15.89	24.66	20.58	15.54	15.00	0.02	-0.33	-0.19	0.66	-0.53	-0.84
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$													
$\widehat{\tau}$	0.50	0.60	0.60	0.50	0.50	0.60	0.50	0.40	0.80	0.20	0.40	0.40	0.40
SupT	2.95*	2.35	1.83	3.69‡	3.94‡	3.47†	4.34‡	2.57	2.27	2.65	2.4	2.43	2.08
AveT	1.72‡	1.36†	.87	2.63‡	2.56‡	2.19‡	2.48‡	1.13	0.97	.92	1.04	1.05	1.08
CD	20.37	16.35	15.98	24.43	20.26	15.81	14.56	0.01	-0.36	1.17	0.60	-0.56	-0.46
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$													
$\widehat{\tau}$	0.60	0.60	0.60	0.50	0.60	0.60	0.60	0.50	0.50	0.50	0.50	0.50	0.50
SupT	4.83‡	5.03‡	4.49‡	4.49‡	4.38‡	3.84‡	4.91‡	4.01†	4.15†	3.11*	3.66†	4.2†	3.44
AveT	3.56‡	3.34‡	2.99‡	3.41‡	3.17‡	2.75‡	3.19‡	2.03‡	2.25‡	1.91‡	2.09‡	2.16‡	2.38‡
CD	20.80	16.32	15.89	25.37	21.12	16.04	15.64	-0.16	-0.38	0.10	1.09	-0.30	-0.68

Notes: In addition to Δd_{it}, inflation (π_{it}) and its lagged values are included as regressors in the ARDL and DL specifications, (22)–(23), while the CS-ARDL and CS-DL specifications, (24)–(25), also include the cross-sectional averages of π_{it} and its lagged values. See also the notes to Table 4.

Table 8: Mean group estimates of the long-run effects of public debt on output growth (1966-2010)

	ARDL			DL				CS-ARDL			CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	(3,3,3)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$														
All Countries	-0.058‡	-0.044‡	-0.048‡	-0.084‡	-0.073‡	-0.057‡	-0.056‡	-0.069‡	-0.078‡	-0.100‡	-0.068‡	-0.073‡	-0.071‡	-0.061‡
	(0.0133)	(0.0129)	(0.0144)	(0.0098)	(0.0107)	(0.0117)	(0.0125)	(0.0111)	(0.013)	(0.0294)	(0.0112)	(0.0099)	(0.0128)	(.00165)
Advanced Economies	-0.041	-0.027	-0.024	-0.105‡	-0.079‡	-0.062‡	-0.039†	-0.047†	-0.060‡	-0.042†	-0.079‡	-0.065‡	-0.053†	-0.032*
	(0.0253)	(0.0242)	(0.0236)	(0.0142)	(0.0165)	(0.017)	(0.0174)	(0.02)	(0.0223)	(0.0174)	(0.0216)	(0.0156)	(00232)	(0.0182)
Developing Economies	-0.066‡	-0.049‡	-0.056‡	-0.067‡	-0.075‡	-0.049‡	-0.071‡	-0.072‡	-0.076‡	-0.771	-0.058‡	-0.068‡	-0.072‡	-0.071‡
	(0.0129)	(0.0139)	(0018)	(0.0127)	(0.0132)	(0.0133)	(0.0153)	(0.012)	(0.0131)	(0.6541)	(0.0147)	(0.0129)	(0.0142)	(0.0254)
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$														
All Countries	-0.072‡	-0.059‡	-0.062‡	-0.095‡	-0.084‡	-0.072‡	-0.074‡	-0.087‡	-0.095‡	-0.127‡	-0.084‡	-0.092‡	-0.092‡	-0.083‡
	(0.0128)	(0.0121)	(0.0141)	(0.011)	(0.0111)	(0.0112)	(0.0121)	(0.0121)	(0.0133)	(0.0265)	(0.0107)	(0.0113)	(0.0133)	(0.0163)
Advanced Economies	-0.070‡	-0.062‡	-0.054‡	-0.113‡	-0.086‡	-0.070‡	-0.055‡	-0.094‡	-0.110‡	-0.101‡	-0.095‡	-0.104‡	-0.103‡	-0.100‡
	(0.0228)	(0.0193)	(0.0203)	(0.0161)	(0.018)	(0.0173)	(0.016)	(0.0199)	(0.0217)	(0.0253)	(0.0141)	(0.0189)	(0.0214)	(0.0208)
Developing Economies	-0.076‡	-0.061‡	-0.072‡	-0.076‡	-0.084‡	-0.079‡	-0.090‡	-0.082‡	-0.082‡	-0.146‡	-0.073‡	-0.080‡	-0.076‡	-0.063‡
	(0.0124)	(0.0125)	(0.0176)	(0.0136)	(0.0127)	(0.0122)	(0.0151)	(0.0129)	(0.0135)	(0.0501)	(0.0163)	(0.0133)	(0.0139)	(0.0241)
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$														
All Countries	-0.057‡	-0.043‡	-0.048‡	-0.079‡	-0.066‡	-0.049‡	-0.041‡	-0.078‡	-0.082‡	-0.084‡	-0.076‡	-0.079‡	-0.074‡	-0.062‡
	(0.0134)	(0.0131)	(0.0144)	(0.0097)	(0.011)	(0.0121)	(0.0135)	(0.0112)	(0.013)	(0.02)	(0.0108)	(0.0106)	(0.013)	(0.0142)
Advanced Economies	-0.047*	-0.032	-0.030	-0.097‡	-0.068‡	-0.039*	-0.041†	-0.043†	-0.047†	-0.035*	-0.082‡	-0.052‡	-0.043*	-0.022
	(0.0244)	(0.0229)	(0.0225)	(0.0147)	(0.0177)	(0.0206)	(0.0176)	(0.0177)	(0.0204)	(0.0187)	(0.0136)	(0.0165)	(0.0224)	(0.0203)
Developing Economies	-0.065‡	-0.048‡	-0.057‡	-0.068‡	-0.070‡	-0.053‡	-0.051‡	-0.077‡	-0.071‡	3.534	-0.069‡	-0.072‡	-0.066‡	-0.056†
	(0.013)	(0.0139)	(0.018)	(0.0129)	(0.0132)	(0.0139)	(0.0166)	(0.0129)	(0.0132)	(3.5413)	(0.0161)	(0.0131)	(0.0142)	(0.0223)
(d) Regressions without threshold variables														
All Countries	-0.070‡	-0.061‡	-0.066‡	-0.083‡	-0.070‡	-0.065‡	-0.064‡	-0.082‡	-0.086‡	-0.096‡	-0.085‡	-0.080‡	-0.068‡	-0.060‡
	(0.015)	(0.014)	(0.016)	(0.012)	(0.012)	(0.012)	(0.014)	(0.012)	(0.014)	(0.016)	(0.012)	(0.013)	(0.014)	(0.017)
Advanced Economies	-0.060†	-0.049*	-0.043	-0.086‡	-0.067‡	-0.055‡	-0.050‡	-0.081‡	-0.093‡	-0.092‡	-0.094‡	-0.093‡	-0.081‡	-0.067‡
	(0.028)	(0.026)	(0.027)	(0.020)	(0.021)	(0.019)	(0.019)	(0.021)	(0.024)	(0.026)	(0.019)	(0.023)	(0.020)	(0.019)
Developing Economies	-0.079‡	-0.071‡	-0.086‡	-0.081‡	-0.072‡	-0.074‡	-0.077‡	-0.082‡	-0.080‡	-0.099‡	-0.077‡	-0.069‡	-0.057‡	-0.053*
	(0.013)	(0.013)	(0.018)	(0.013)	(0.013)	(0.016)	(0.019)	(0.013)	(0.014)	(0.020)	(0.014)	(0.015)	(0.021)	(0.028)

Notes: The ARDL and DL specifications are given by (22) and (23) while the CS-ARDL and CS-DL specifications are given by (24) and (25). Standard errors are given in parentheses. Statistical significance is denoted by *, † and ‡, at 10%, 5% and 1% level, respectively.

Table 9: Mean group estimates of the long-run effects of public debt and inflation on output growth for *all countries*, 1966-2010

	ARDL			DL				CS-ARDL		CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$													
$\widehat{\phi}_{\Delta d}$	-0.052‡	-0.053‡	-0.061‡	-0.085‡	-0.069‡	-0.065‡	-0.059‡	-0.077‡	-0.088‡	-0.072‡	-0.079‡	-0.078‡	-0.073‡
	(0.011)	(0.013)	(0.014)	(0.010)	(0.010)	(0.011)	(0.013)	(0.012)	(0.016)	(0.011)	(0.011)	(0.014)	(0.021)
$\widehat{\phi}_{\pi}$	-0.068*	-0.007	0.019	-0.042†	-0.049†	-0.002	-0.006	-0.138‡	-0.137‡	-0.130‡	-0.135‡	-0.152‡	-0.193‡
	(0.021)	(0.025)	(0.036)	(0.020)	(0.020)	(0.025)	(0.025)	(0.027)	(0.039)	(0.024)	(0.027)	(0.040)	(0.049)
CD	20.36	16.33	15.89	24.66	20.58	15.54	15.00	0.02	-0.33	-0.19	0.66	-0.53	-0.84
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$													
$\widehat{\phi}_{\Delta d}$	-0.068‡	-0.070‡	-0.077‡	-0.096‡	-0.081‡	-0.080‡	-0.083‡	-0.091‡	-0.100‡	-0.088‡	-0.095‡	-0.096‡	-0.087‡
	(0.011)	(0.013)	(0.014)	(0.011)	(0.010)	(0.012)	(0.013)	(0.014)	(0.017)	(0.011)	(0.013)	(0.015)	(0.020)
$\widehat{\phi}_{\pi}$	-0.063‡	0.000	0.027	-0.038*	-0.044†	0.000	0.003	-0.141‡	-0.149‡	-0.099‡	-0.134‡	-0.150‡	-0.197‡
	(0.021)	(0.026)	(0.038)	(0.022)	(0.021)	(0.026)	(0.025)	(0.033)	(0.045)	(0.023)	(0.031)	(0.049)	(0.061)
CD	20.37	16.35	15.98	24.43	20.26	15.81	14.56	0.01	-0.36	1.17	0.60	-0.56	-0.46
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$													
$\widehat{\phi}_{\Delta d}$	-0.054‡	-0.053‡	-0.060‡	-0.081‡	-0.063‡	-0.061‡	-0.049‡	-0.078‡	-0.079‡	-0.080‡	-0.082‡	-0.077‡	-0.069‡
	(0.012)	(0.013)	(0.014)	(0.010)	(0.010)	(0.011)	(0.014)	(0.012)	(0.016)	(0.011)	(0.011)	(0.014)	(0.020)
$\widehat{\phi}_{\pi}$	-0.060‡	-0.008	0.015	-0.036*	-0.042†	0.002	0.001	-0.151‡	-0.137‡	-0.116‡	-0.120‡	-0.131‡	-0.134‡
	(0.021)	(0.025)	(0.036)	(0.021)	(0.020)	(0.025)	(0.025)	(0.028)	(0.035)	(0.023)	(0.025)	(0.036)	(0.046)
CD	20.80	16.32	15.89	25.37	21.12	16.04	15.64	-0.16	-0.38	0.10	1.09	-0.30	-0.68
(d) Regressions without threshold variables													
$\widehat{\phi}_{\Delta d}$	-0.070‡	-0.076‡	-0.083‡	-0.080‡	-0.082‡	-0.077‡	-0.070‡	-0.085‡	-0.090‡	-0.090‡	-0.091‡	-0.082‡	-0.060‡
	(0.012)	(0.013)	(0.014)	(0.010)	(0.012)	(0.013)	(0.013)	(0.014)	(0.016)	(0.013)	(0.016)	(0.020)	(0.022)
$\widehat{\phi}_{\pi}$	-0.038*	0.021	0.040	-0.017	0.026	0.044	0.036	-0.110‡	-0.097‡	-0.075‡	-0.080†	-0.086†	-0.124‡
	(0.023)	(0.030)	(0.040)	(0.023)	(0.030)	(0.031)	(0.032)	(0.028)	(0.034)	(0.024)	(0.035)	(0.040)	(0.047)
CD	21.39	16.63	15.98	22.07	16.83	16.42	16.13	-0.13	-0.44	0.97	0.45	0.63	3.16

Notes: In addition to Δd_{it}, inflation (π_{it}) and its lagged values are included as regressors in the ARDL and DL specifications, (22)–(23), while the CS-ARDL and CS-DL specifications, (24)–(25), also include the cross-sectional averages of π_{it} and its lagged values. Statistical significance is denoted by $*$, $\dagger$ and $\ddagger$, at 10%, 5% and 1% level, respectively.

A Data Appendix

Output growth is computed using real gross domestic product (GDP) data series obtained from the International Monetary Fund *International Financial Statistics* database. The gross government deb-to-GDP data series for the majority of the countries are downloaded from http://www.carmenreinhart.com/data/browse-by-topic/topics/9/ which are the updates of those discussed in Reinhart and Rogoff (2011). For Iran, Morocco, Nigeria, and Syria the debt-to-GDP series are obtained from the International Monetary Fund FAD *Historical Public Debt* database. We focus on gross debt data due to difficulty of collecting net debt data on a consistent basis over time and across countries. Moreover, we use public debt at the general government level for as many countries as possible (Austria, Belgium, Germany, Italy, Netherlands, New Zealand, Singapore, Spain, Sweden, and Tunisia), but given the lack of general public debt data for many countries, central government debt data is used as an alternative.[10]

Price inflation data are computed using the consumer price index (CPI) obtained from the International Monetary Fund *International Financial Statistics* database, except for the CPI data for Brazil, China and Tunisia which are obtained from the International Monetary Fund, *World Economic Outlook* database, and the CPI data for the UK, which is obtained from the Reinhart and Rogoff (2010) *Growth in a Time of Debt* database.

Table A.1: List of the 40 countries in the sample

Europe	**MENA Countries**	**Asia Pacific**	**Latin America**
Austria*	Egypt	Australia*	Argentina
Belgium*	Iran	China	Brazil
Finland*	Morocco	India	Chile
France*	Syria	Indonesia	Ecuador
Germany*	Tunisia	Japan*	Peru
Italy*	Turkey	Korea*	Venezuela
Netherlands*		Malaysia	
Norway*	**North America**	New Zealand*	**Rest of Africa**
Spain*	Canada*	Philippines	Nigeria
Sweden*	Mexico	Singapore*	South Africa
Switzerland*	United States*	Thailand	
United Kingdom*			

Notes: * indicates that the country is classified as an advanced economy, as defined by the International Monetary Fund.

[10] The complete dataset, Matlab codes, and Stata do files needed to generate the empirical results in this paper are available from people.ds.cam.ac.uk/km418.

References

Andrews, D. W. K. and W. Ploberger (1994). Optimal Tests when a Nuisance Parameter is Present Only Under the Alternative. *Econometrica 62*(6), pp. 1383–1414.

Bai, J. (2009). Panel Data Models with Interactive Fixed Effects. *Econometrica 77*, 1229–1279.

Checherita-Westphal, C. and P. Rother (2012). The impact of High Government Debt on Economic Growth and its Channels: An Empirical Investigation for the Euro Area. *European Economic Review 56*(7), 1392 – 1405.

Chudik, A., K. Mohaddes, M. H. Pesaran, and M. Raissi (2013). Debt, Inflation and Growth: Robust Estimation of Long-Run Effects in Dynamic Panel Data Models. *Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute Working Paper No. 162*.

Chudik, A., K. Mohaddes, M. H. Pesaran, and M. Raissi (2015). Long-Run Effects in Large Heterogeneous Panel Data Models with Cross-Sectionally Correlated Errors. Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute Working Paper No. 223.

Chudik, A. and M. H. Pesaran (2015a). Common Correlated Effects Estimation of Heterogeneous Dynamic Panel Data Models with Weakly Exogenous Regressors. *Journal of Econometrics forthcoming*.

Chudik, A. and M. H. Pesaran (2015b). Large Panel Data Models with Cross-Sectional Dependence: A Survey. In B. H. Baltagi (Ed.), *The Oxford Handbook of Panel Data*, pp. 3–45. Oxford University Press, New York.

Davies, R. B. (1977). Hypothesis Testing when a Nuisance Parameter is Present only under the Alternative. *Biometrika 64*(2), 247–254.

Davies, R. B. (1987). Hypothesis Testing when a Nuisance Parameter is Present Only Under the Alternatives. *Biometrika 74*(1), pp. 33–43.

Eberhardt, M. and A. F. Presbitero (2015). Public Debt and Growth: Heterogeneity and Nonlinearity. *forthcoming in Journal of International Economics*.

Ghosh, A. R., J. I. Kim, E. G. Mendoza, J. D. Ostry, and M. S. Qureshi (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *The Economic Journal 123*(566), F4–F30.

Hansen, B. E. (1996). Inference When a Nuisance Parameter Is Not Identified Under the Null Hypothesis. *Econometrica 64*(2), pp. 413–430.

Hansen, B. E. (1999). Threshold Effects in Non-Dynamic Panels: Estimation, Testing, and Inference. *Journal of Econometrics 93*(2), 345 – 368.

Krugman, P. (1988). Financing vs. Forgiving a Debt Overhang. *Journal of Development Economics 29*(3), 253–268.

Moon, H. R. and M. Weidner (2014). Dynamic Linear Panel Regression Models with Interactive Fixed Effects. *CeMMaP Working Paper CWP47/14*.

Panizza, U. and A. F. Presbitero (2013). Public Debt and Economic Growth in Advanced Economies: A Survey. *Swiss Journal of Economics and Statistics 149* (II), 175–204.

Pesaran, M. H. (1997). The Role of Economic Theory in Modelling the Long Run. *Economic Journal 107*, 178–191.

Pesaran, M. H. (2004). General Diagnostic Tests for Cross Section Dependence in Panels. *IZA Discussion Paper No. 1240*.

Pesaran, M. H. (2015). Testing Weak Cross-Sectional Dependence in Large Panels. *Econometric Reviews 34* (6-10), 1089–1117.

Pesaran, M. H. and Y. Shin (1999). An Autoregressive Distributed Lag Modelling Approach to Cointegration Analysis. In S. Strom (Ed.), *Econometrics and Economic Theory in 20th Century: The Ragnar Frisch Centennial Symposium*, Chapter 11, pp. 371–413. Cambridge: Cambridge University Press.

Pesaran, M. H. and R. Smith (1995). Estimating Long-run Relationships from Dynamic Heterogeneous Panels. *Journal of Econometrics 68* (1), 79–113.

Reinhart, C. M., V. R. Reinhart, and K. S. Rogoff (2012). Public Debt Overhangs: Advanced-Economy Episodes Since 1800. *The Journal of Economic Perspectives 26* (3), 69–86.

Reinhart, C. M. and K. S. Rogoff (2010). Growth in a Time of Debt. *American Economic Review 100* (2), 573–78.

Reinhart, C. M. and K. S. Rogoff (2011). From Financial Crash to Debt Crisis. *American Economic Review 101* (5), 1676–1706.

Seo, M. H. and Y. Shin (2014). Dynamic Panels with Threshold Effect and Endogeneity. *STICERD - Econometrics Paper Series /2014/577*.

Song, M. (2013). Asymptotic Theory for Dynamic Heterogeneous Panels with Cross-Sectional Dependence and Its Applications. Mimeo, January 2013.

Woo, J. and M. S. Kumar (2015). Public Debt and Growth. *Economica forthcoming*.

The Supplement to:
"Is There a Debt-threshold Effect on Output Growth?"

Alexander Chudik[a], Kamiar Mohaddes[b], M. Hashem Pesaran[c], and Mehdi Raissi[d]

[a] Federal Reserve Bank of Dallas, USA

[b] Faculty of Economics and Girton College, University of Cambridge, UK

[c] Department of Economics, University of Southern California, USA
and Trinity College, Cambridge, UK

[d] International Monetary Fund, Washington DC, USA

July 3, 2015

Abstract

This document provides supplemental results for the paper Chudik, Mohaddes, Pesaran and Raissi (2015) "Is There a Debt-threshold Effect on Output Growth?". It presents a complete set of Monte Carlo findings for the experiments outlined in the paper (Part A) and additional empirical findings (Part B).

Part A: Monte Carlo Results

1 Introduction

The Monte Carlo set-up is described in Section 3.2 of Chudik, Mohaddes, Pesaran, and Raissi (2015), hereafter CMPR, and covers 6 different experimental designs. For each design, the performance of the *Sup* and *Ave* tests of the threshold effects ($\varphi_1 = 0$ and/or $\varphi_2 = 0$), and the small sample performance of the filtered-pooled estimators of the threshold coefficients (φ_1 and/or φ_2) as well as the threshold level, τ, are investigated. DGPs outlined in CMPR imply the following probabilities and correlations:

	DGP1	DGP2	DGP3	DGP4	DGP5	DGP6
	($\varphi_1 = 0.01$)	($\varphi_1 = 0.01$)	($\varphi_1 = 0.01$)	($\varphi_1 = 0$)	($\varphi_1 = 0.01$)	($\varphi_1 = 0$)
	($\varphi_2 = 0$)	($\varphi_2 = 0$)	($\varphi_2 = 0$)	($\varphi_2 = 0.01$)	($\varphi_2 = 0$)	($\varphi_2 = 0$)
$P\left[g_1\left(d_{it},\tau\right)=1\right]$	37%	37%	37%	-	38%	41%
$P\left[g_2\left(d_{it},\tau\right)=1\right]$	-	-	-	23%	-	-
Correlation between $g_1\left(d_{it},\tau\right)$ and e_{it}	39%	16%	17%	-	14%	27%
Correlation between $g_2\left(d_{it},\tau\right)$ and e_{it}	-	-	-	27%		-
Correlation between f_{1t} and $g_1\left(d_{it},\tau\right)$	-	-	-	-	0%	-17%
$E\left\|\lambda_1\left(\mathbf{\Psi}_i\right)\right\|$	0	0.90	0.90	0.90	0.90	0
R^2 (output equation)	9%	50%	47%	51%	65%	50%

We refer the reader to CMPR for detailed description of the design and the objectives of these experiments. The next section presents the MC findings.

2 Monte Carlo Tables

Table 1: MC findings for Bias(x100) and RMSE(x100) of the estimation of φ_1 and τ in DGP1

	Pooled estimator				**Fixed Effects estimator**				**Filtered Pooled estimator**			
	Bias (x100)		RMSE (x100)		Bias (x100)		RMSE (x100)		Bias (x100)		RMSE (x100)	
(N,T)	**46**	**100**	**46**	**100**	**46**	**100**	**46**	**100**	**46**	**100**	**46**	**100**
	φ_1 (true value $= -0.01$)											
40	1.5137	1.5117	1.5190	1.5152	1.5410	1.5415	1.5464	1.5450	0.0172	0.0065	0.1416	0.0962
100	1.5095	1.5091	1.5117	1.5105	1.5399	1.5395	1.5421	1.5409	0.0054	0.0011	0.0909	0.0614
	τ (true value $= 0.80$)											
40	-74.36	-74.73	74.37	74.73	-74.52	-74.79	74.52	74.79	0.00	0.00	1.55	0.70
100	-74.77	-74.89	74.77	74.89	-74.80	-74.93	74.80	74.93	-0.02	0.00	0.61	0.25

Notes: Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it})'$ as the vector of filtering variables.

Table 2a: Rejection frequencies of tests of $\varphi_1 = 0$ in DGP1, computed based on the pooled estimator of φ_1

	Rejection rates (x100):									
	$Sup\mathcal{T}$		$Ave\mathcal{T}$		$\mathcal{T}\,(0.2)$		$\mathcal{T}\,(0.5)$		$\mathcal{T}(0.9)$	
(N,T)	46	100	46	100	46	100	46	100	46	100
	$\varphi_1 = -0.01$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.009$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.008$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.007$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.006$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.005$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.004$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.003$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.002$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.001$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.001$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.002$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.003$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.004$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.005$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.006$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.007$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.008$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.009$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.01$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Notes: $Sup\mathcal{T}$ and $Ave\mathcal{T}$ are Sup and Ave, t-tests of $\varphi_1 = 0$ in DGP1, with rejection frequencies computed at $\varphi_1 = -0.01, 0.009, ..., 0.0, 0.001, ..., 0.009, 0.01$. $\mathcal{T}\,(\tau)$ is the t-test of the threshold effect ($\varphi_1 = 0$) computed for three *a priori* selected values of τ, $\tau = 0.2, 0.5$ and 0.9.

Table 2b: Rejection frequencies of tests of $\varphi_1 = 0$ in DGP1, computed based on the fixed effect estimator of φ_1

	Rejection rates (x100):									
	SupT		*AveT*		$\mathcal{T}(0.2)$		$\mathcal{T}(0.5)$		$\mathcal{T}(0.9)$	
(N,T)	46	100	46	100	46	100	46	100	46	100
	$\varphi_1 = -0.01$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.009$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.008$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.007$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.006$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.005$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.004$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.003$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.002$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = -0.001$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.001$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.002$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.003$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.004$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.005$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.006$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.007$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.008$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.009$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
	$\varphi_1 = 0.01$									
40	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00

Notes: See notes to Table 2a.

Table 2c: Rejection frequencies of tests of $\varphi_1 = 0$ in DGP1, computed based on the filtered pooled estimator of φ_1

	Rejection rates (x100):									
	SupT		*AveT*		$\mathcal{T}$ (0.2)		$\mathcal{T}$ (0.5)		$\mathcal{T}$ (0.9)	
(N,T)	46	100	46	100	46	100	46	100	46	100
	$\varphi_1 = -0.01$									
40	100.00	100.00	99.95	100.00	77.85	98.05	24.25	46.25	99.90	100.00
100	100.00	100.00	100.00	100.00	99.30	100.00	49.30	83.65	100.00	100.00
	$\varphi_1 = -0.009$									
40	99.95	100.00	99.85	100.00	68.50	95.75	20.50	38.70	99.70	100.00
100	100.00	100.00	100.00	100.00	97.40	100.00	41.15	75.80	100.00	100.00
	$\varphi_1 = -0.008$									
40	99.75	100.00	99.50	100.00	61.80	89.90	17.00	33.50	98.90	100.00
100	100.00	100.00	100.00	100.00	92.85	99.90	32.90	68.95	100.00	100.00
	$\varphi_1 = -0.007$									
40	98.65	100.00	97.10	100.00	48.65	81.00	13.85	26.60	96.10	100.00
100	100.00	100.00	100.00	100.00	86.50	99.45	27.25	56.25	100.00	100.00
	$\varphi_1 = -0.006$									
40	94.70	100.00	90.55	99.75	38.45	68.30	12.50	20.40	89.20	99.75
100	100.00	100.00	100.00	100.00	74.10	97.10	21.75	43.30	99.85	100.00
	$\varphi_1 = -0.005$									
40	83.70	99.40	79.75	98.20	29.15	52.85	9.95	16.35	78.85	97.45
100	99.85	100.00	99.35	100.00	58.20	88.50	16.05	32.45	99.00	100.00
	$\varphi_1 = -0.004$									
40	62.30	93.40	61.35	90.65	20.40	37.85	8.80	13.20	59.05	89.35
100	96.70	100.00	94.05	100.00	41.70	74.85	12.60	22.95	92.70	99.85
	$\varphi_1 = -0.003$									
40	36.35	71.60	38.35	68.00	14.10	23.00	6.95	7.85	37.80	67.50
100	77.50	98.85	75.00	97.85	25.10	50.45	9.00	16.20	73.45	97.25
	$\varphi_1 = -0.002$									
40	18.70	35.25	19.90	37.40	9.85	13.25	6.35	7.00	20.60	38.25
100	39.45	75.10	40.05	72.90	14.00	24.70	7.10	9.85	40.25	70.90
	$\varphi_1 = -0.001$									
40	9.85	11.10	10.85	11.90	7.70	8.00	6.35	5.45	10.00	12.45
100	13.25	24.05	15.75	25.35	7.75	10.60	6.15	6.35	14.85	24.95
	$\varphi_1 = 0$	(size)								
40	6.30	5.00	6.10	5.60	5.35	4.85	5.40	6.10	4.45	4.80
100	5.30	5.50	5.45	6.35	6.00	6.05	4.50	5.45	5.55	6.15
	$\varphi_1 = 0.001$									
40	8.55	11.25	9.20	12.70	7.10	6.35	4.90	5.40	7.90	13.00
100	12.60	23.40	15.05	25.70	7.20	10.20	5.90	6.60	14.60	25.55
	$\varphi_1 = 0.002$									
40	16.10	34.65	18.30	37.25	9.30	13.60	5.00	6.70	18.05	36.75
100	39.65	76.45	40.95	73.50	13.50	26.25	6.35	9.70	40.60	72.50
	$\varphi_1 = 0.003$									
40	38.20	70.85	38.50	69.65	13.70	24.00	6.90	9.30	39.55	66.40
100	77.85	98.90	74.65	97.50	26.15	49.85	9.50	15.35	74.15	97.20
	$\varphi_1 = 0.004$									
40	61.35	94.35	60.15	91.70	19.70	36.45	8.70	11.15	58.75	90.70
100	96.85	100.00	93.75	99.95	43.05	73.65	12.85	22.80	93.30	100.00
	$\varphi_1 = 0.005$									
40	82.30	99.20	79.75	98.05	29.80	51.35	10.85	16.95	78.00	97.70
100	99.80	100.00	99.35	100.00	61.70	89.70	16.90	31.75	99.30	100.00
	$\varphi_1 = 0.006$									
40	94.70	100.00	92.30	99.95	39.45	68.10	12.85	21.40	91.50	99.90
100	100.00	100.00	99.90	100.00	75.60	97.05	24.35	41.55	99.95	100.00
	$\varphi_1 = 0.007$									
40	98.35	100.00	97.20	100.00	50.05	81.25	14.65	27.75	96.05	99.95
100	100.00	100.00	100.00	100.00	86.05	99.10	28.10	55.45	100.00	100.00
	$\varphi_1 = 0.008$									
40	99.80	100.00	99.30	100.00	59.45	88.80	17.20	31.55	99.00	100.00
100	100.00	100.00	100.00	100.00	93.75	99.95	35.20	66.10	100.00	100.00
	$\varphi_1 = 0.009$									
40	100.00	100.00	99.75	100.00	68.00	95.65	19.85	39.15	99.65	100.00
100	100.00	100.00	100.00	100.00	97.25	100.00	41.45	78.20	100.00	100.00
	$\varphi_1 = 0.01$									
40	100.00	100.00	100.00	100.00	78.05	98.40	23.85	46.45	100.00	100.00
100	100.00	100.00	100.00	100.00	98.80	100.00	50.15	83.90	100.00	100.00

Notes: Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it})'$ as the vector of filtering variables. See notes to Table 2a.

Table 3: MC findings for Bias(x100) and RMSE(x100) of the estimation of φ_1 and τ in DGP2

	Filtered pooled estimator			
	Bias (x100)		RMSE (x100)	
(N,T)	**40**	**100**	**40**	**100**
	φ_1 (true value $= -0.01$)			
40	0.0310	0.0164	0.1434	0.0875
100	0.0241	0.0145	0.0944	0.0570
	τ (true value $= 0.80$)			
40	-0.05	-0.01	1.06	0.40
100	0.00	0.00	0.39	0.16

Notes: Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it}, d_{i,t-1}, d_{i,t-2}, \Delta y_{i,t-1})'$ as the vector of filtering variables.

Table 4: Rejection frequencies of tests of $\varphi_1 = 0$ in DGP2, computed based on the filtered pooled estimator of φ_1

	Rejection rates (x100):									
	Sup$\mathcal{T}$		*Ave*$\mathcal{T}$		$\mathcal{T}$ (0.2)		$\mathcal{T}$ (0.5)		$\mathcal{T}$ (0.9)	
(N,T)	46	100	46	100	46	100	46	100	46	100
	$\varphi_1 = -0.01$									
40	100.00	100.00	100.00	100.00	58.45	97.40	12.75	35.40	99.95	100.00
100	100.00	100.00	100.00	100.00	91.05	100.00	21.05	65.65	100.00	100.00
	$\varphi_1 = -0.009$									
40	100.00	100.00	99.95	100.00	52.50	93.50	11.35	29.65	99.65	100.00
100	100.00	100.00	100.00	100.00	86.10	99.95	17.05	59.20	100.00	100.00
	$\varphi_1 = -0.008$									
40	100.00	100.00	99.80	100.00	44.80	87.60	10.10	26.50	99.30	100.00
100	100.00	100.00	100.00	100.00	79.10	99.85	14.50	49.95	100.00	100.00
	$\varphi_1 = -0.007$									
40	99.35	100.00	98.00	100.00	37.25	79.50	10.50	22.30	95.80	100.00
100	100.00	100.00	100.00	100.00	67.10	99.15	13.55	41.35	100.00	100.00
	$\varphi_1 = -0.006$									
40	96.65	100.00	93.00	100.00	29.35	67.50	9.40	16.70	88.20	99.95
100	100.00	100.00	100.00	100.00	55.10	96.40	12.85	32.50	99.90	100.00
	$\varphi_1 = -0.005$									
40	88.85	99.95	83.70	99.40	23.90	53.95	9.30	13.80	77.65	99.45
100	99.90	100.00	99.45	100.00	45.55	88.60	11.80	25.10	98.40	100.00
	$\varphi_1 = -0.004$									
40	68.05	98.30	64.50	96.25	18.20	39.55	8.95	11.75	59.10	94.20
100	97.35	100.00	94.35	100.00	30.85	72.70	8.95	18.20	92.90	100.00
	$\varphi_1 = -0.003$									
40	45.65	84.95	44.25	81.35	12.70	26.00	6.75	9.10	39.90	76.95
100	84.50	99.65	80.05	99.10	21.00	49.00	9.20	14.00	75.25	98.55
	$\varphi_1 = -0.002$									
40	22.70	47.30	24.75	47.25	10.35	14.95	7.70	7.30	21.50	46.15
100	49.35	89.30	49.65	84.20	13.50	27.55	8.50	10.10	44.35	81.20
	$\varphi_1 = -0.001$									
40	11.95	13.80	12.60	16.30	6.90	8.00	7.95	6.10	11.30	15.30
100	19.10	31.40	20.10	32.55	9.45	11.00	7.80	7.45	17.50	31.80
	$\varphi_1 = 0$									
40	8.75	7.25	8.20	6.15	6.70	5.80	6.25	5.55	7.35	5.75
100	8.30	6.55	8.15	6.95	7.35	6.70	7.40	6.15	7.70	5.60
	$\varphi_1 = 0.001$									
40	11.95	15.15	12.45	17.15	8.20	7.70	7.25	5.75	11.30	15.60
100	16.95	30.40	18.95	31.95	9.40	11.55	7.25	6.65	16.40	30.00
	$\varphi_1 = 0.002$									
40	21.45	45.50	23.55	45.45	9.50	13.50	8.30	6.75	21.15	43.65
100	46.05	87.90	45.90	82.45	13.60	25.50	7.35	9.75	41.85	81.40
	$\varphi_1 = 0.003$									
40	42.05	83.75	42.80	78.95	13.75	24.45	8.10	8.15	38.75	75.85
100	81.80	99.75	76.95	98.90	21.80	51.10	8.40	12.15	71.20	98.80
	$\varphi_1 = 0.004$									
40	66.90	98.15	64.00	95.35	17.35	37.25	7.95	11.10	59.60	94.00
100	97.65	100.00	94.80	100.00	32.35	73.30	9.70	17.65	91.25	99.95
	$\varphi_1 = 0.005$									
40	86.60	100.00	80.20	99.50	23.85	54.25	8.15	13.20	76.75	99.10
100	99.70	100.00	99.10	100.00	44.75	89.50	10.15	26.05	98.10	100.00
	$\varphi_1 = 0.006$									
40	95.85	100.00	92.60	100.00	29.60	67.80	9.00	17.15	87.90	99.95
100	100.00	100.00	100.00	100.00	55.85	96.70	12.00	31.65	99.95	100.00
	$\varphi_1 = 0.007$									
40	99.05	100.00	98.10	100.00	38.15	81.30	9.75	20.55	96.10	100.00
100	100.00	100.00	100.00	100.00	67.85	99.35	14.45	40.00	100.00	100.00
	$\varphi_1 = 0.008$									
40	99.95	100.00	99.50	100.00	46.10	88.65	10.50	23.35	98.70	100.00
100	100.00	100.00	100.00	100.00	78.80	99.75	16.20	49.10	100.00	100.00
	$\varphi_1 = 0.009$									
40	99.95	100.00	99.80	100.00	52.70	94.10	11.55	29.70	99.45	100.00
100	100.00	100.00	100.00	100.00	86.45	100.00	16.90	59.30	100.00	100.00
	$\varphi_1 = 0.01$									
40	100.00	100.00	100.00	100.00	58.35	96.80	12.70	34.10	99.90	100.00
100	100.00	100.00	100.00	100.00	92.35	100.00	21.85	64.45	100.00	100.00

Notes: *Sup*$\mathcal{T}$ and *Ave*$\mathcal{T}$ are *Sup* and *Ave*, t-tests of $\varphi_1 = 0$ in DGP2, with rejection frequencies computed at $\varphi_1 = -0.01, 0.009, ..., 0.0, 0.001, ..., 0.009, 0.01$. $\mathcal{T}(\tau)$ is the t-test of the threshold effect ($\varphi_1 = 0$) computed for three *a priori* selected values of τ, $\tau = 0.2, 0.5$ and 0.9. Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it}, d_{i,t-1}, d_{i,t-2}, \Delta y_{i,t-1})'$ as the vector of filtering variables.

Table 5: MC findings for Bias(x100) and RMSE(x100) of the estimation of φ_1 and τ in DGP3

	Filtered pooled estimator			
	Bias (x100)		RMSE (x100)	
(N,T)	**40**	**100**	**40**	**100**
	φ_1 (true value $= -0.01$)			
40	0.0225	0.0199	0.1403	0.0855
100	0.0241	0.0148	0.0908	0.0553
	τ (true value $= 0.80$)			
40	-0.07	-0.02	0.89	0.32
100	0.00	0.00	0.34	0.13

Notes: Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it}, d_{i,t-1}, d_{i,t-2}, \Delta y_{i,t-1})'$ as the vector of filtering variables.

Table 6: Rejection frequencies of tests of $\varphi_1 = 0$ in DGP3, computed based on the filtered pooled estimator of φ_1

	Rejection rates (x100):									
	SupT		*AveT*		$\mathcal{T}$ (0.2)		$\mathcal{T}$ (0.5)		$\mathcal{T}$ (0.9)	
(N,T)	46	100	46	100	46	100	46	100	46	100
	$\varphi_1 = -0.01$									
40	100.00	100.00	100.00	100.00	62.10	98.40	16.35	42.05	100.00	100.00
100	100.00	100.00	100.00	100.00	92.80	100.00	24.20	74.80	100.00	100.00
	$\varphi_1 = -0.009$									
40	100.00	100.00	99.90	100.00	54.80	95.75	13.55	35.60	99.65	100.00
100	100.00	100.00	100.00	100.00	88.10	100.00	21.35	66.45	100.00	100.00
	$\varphi_1 = -0.008$									
40	99.95	100.00	99.30	100.00	47.15	91.25	11.75	30.45	99.15	100.00
100	100.00	100.00	100.00	100.00	80.15	99.90	19.00	58.40	100.00	100.00
	$\varphi_1 = -0.007$									
40	99.50	100.00	98.05	100.00	38.35	83.55	10.15	25.85	97.45	100.00
100	100.00	100.00	100.00	100.00	71.05	99.50	18.25	50.45	100.00	100.00
	$\varphi_1 = -0.006$									
40	97.05	100.00	93.35	100.00	31.80	72.40	10.80	19.30	91.30	100.00
100	100.00	100.00	100.00	100.00	57.55	97.05	13.40	38.40	99.85	100.00
	$\varphi_1 = -0.005$									
40	89.50	99.85	84.65	99.45	25.05	54.90	10.10	16.65	80.45	99.25
100	99.95	100.00	99.65	100.00	43.35	90.25	12.70	30.60	99.05	100.00
	$\varphi_1 = -0.004$									
40	69.95	98.90	67.30	97.10	17.35	42.00	9.10	12.85	62.60	96.20
100	98.25	100.00	96.60	100.00	32.75	76.55	11.35	23.15	93.70	100.00
	$\varphi_1 = -0.003$									
40	45.70	85.75	44.80	81.50	14.85	26.40	8.85	10.10	41.40	78.55
100	85.05	99.90	78.55	99.30	21.15	52.70	8.95	16.40	74.35	99.15
	$\varphi_1 = -0.002$									
40	25.80	49.60	26.35	49.75	9.40	14.00	8.15	8.70	24.50	47.65
100	51.30	89.55	48.00	85.55	14.35	28.30	8.60	9.80	46.95	83.75
	$\varphi_1 = -0.001$									
40	11.00	17.40	12.20	18.65	8.05	8.80	7.80	6.90	10.90	19.60
100	17.55	34.20	18.70	34.95	9.25	11.30	8.50	6.75	18.00	34.95
	$\varphi_1 = 0$									
40	10.50	7.50	9.70	6.95	7.55	6.35	8.50	5.65	7.70	6.15
100	9.65	7.15	9.70	7.25	7.25	7.30	6.95	5.10	7.65	5.85
	$\varphi_1 = 0.001$									
40	12.80	17.00	13.95	19.40	8.15	8.35	8.10	7.35	12.40	18.70
100	18.75	34.05	21.15	34.65	9.05	11.20	7.70	7.90	18.35	34.00
	$\varphi_1 = 0.002$									
40	24.45	52.75	26.35	52.40	10.25	14.70	8.20	9.10	25.45	50.50
100	50.20	90.80	50.20	87.25	14.70	27.40	9.25	11.20	45.00	84.60
	$\varphi_1 = 0.003$									
40	48.00	86.85	46.20	82.45	12.75	24.75	8.45	11.25	43.30	80.80
100	84.55	99.90	79.60	99.35	23.00	52.05	10.25	19.35	76.30	99.30
	$\varphi_1 = 0.004$									
40	71.55	98.90	68.30	97.85	16.80	40.75	9.65	15.05	63.95	95.90
100	98.05	100.00	95.50	100.00	33.70	74.50	12.20	26.30	93.10	100.00
	$\varphi_1 = 0.005$									
40	88.45	99.90	84.25	99.65	22.90	55.50	10.95	19.00	80.10	99.60
100	99.95	100.00	99.55	100.00	43.15	88.95	12.65	38.05	99.15	100.00
	$\varphi_1 = 0.006$									
40	96.95	100.00	93.55	99.95	29.40	68.00	12.30	24.30	90.45	99.90
100	100.00	100.00	99.95	100.00	55.10	96.75	17.40	49.20	100.00	100.00
	$\varphi_1 = 0.007$									
40	99.50	100.00	97.75	100.00	35.45	79.25	13.35	30.30	96.70	100.00
100	100.00	100.00	100.00	100.00	67.00	99.35	20.25	57.65	100.00	100.00
	$\varphi_1 = 0.008$									
40	99.80	100.00	99.55	100.00	42.50	86.70	14.90	38.45	98.90	100.00
100	100.00	100.00	100.00	100.00	74.65	99.65	24.00	70.75	100.00	100.00
	$\varphi_1 = 0.009$									
40	99.95	100.00	99.95	100.00	50.60	93.05	17.05	44.30	99.65	100.00
100	100.00	100.00	100.00	100.00	83.50	99.95	27.30	80.35	100.00	100.00
	$\varphi_1 = 0.01$									
40	100.00	100.00	100.00	100.00	57.85	96.75	18.70	51.00	99.90	100.00
100	100.00	100.00	100.00	100.00	89.65	100.00	33.70	84.85	100.00	100.00

Notes: *SupT* and *AveT* are *Sup* and *Ave*, t-tests of $\varphi_1 = 0$ in DGP3, with rejection frequencies computed at $\varphi_1 = -0.01, 0.009, ..., 0.0, 0.001, ..., 0.009, 0.01$. $\mathcal{T}(\tau)$ is the t-test of the threshold effect ($\varphi_1 = 0$) computed for three *a priori* selected values of τ, $\tau = 0.2, 0.5$ and 0.9. Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it}, d_{i,t-1}, d_{i,t-2}, \Delta y_{i,t-1})'$ as the vector of filtering variables.

Table 7: MC findings for Bias(x100) and RMSE(x100) of the estimation of φ_1,φ_2 and τ in DGP4

	Filtered pooled estimator			
	Bias (x100)		RMSE (x100)	
(N,T)	**40**	**100**	**40**	**100**
	φ_1 (true value = 0)			
40	0.0227	0.0123	0.1489	0.0926
100	0.0190	0.0100	0.1029	0.0600
	φ_2 (true value = -0.01)			
40	-0.0070	-0.0025	0.1362	0.0864
100	-0.0070	-0.0023	0.0891	0.0550
	τ (true value = 0.80)			
40	-0.06	-0.03	1.66	0.65
100	-0.01	-0.01	0.59	0.27

Notes: Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it}, d_{i,t-1}, d_{i,t-2}, \Delta y_{i,t-1})'$ as the vector of filtering variables.

Table 8: Rejection frequencies of tests of $\varphi_1 = \varphi_2 = 0$ in DGP4, computed based on the filtered pooled estimator of φ_1 and φ_2

	Rejection rates (x100):			
	SupF		*AveF*	
(N,T)	46	100	46	100
	$\varphi_1 = 0$ and $\varphi_2 = -0.01$			
40	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = -0.009$			
40	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = -0.008$			
40	99.95	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = -0.007$			
40	99.55	100.00	99.55	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = -0.006$			
40	95.80	100.00	97.00	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = -0.005$			
40	88.75	99.90	90.90	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = -0.004$			
40	68.90	98.55	75.85	98.85
100	98.50	100.00	98.80	100.00
	$\varphi_1 = 0$ and $\varphi_2 = -0.003$			
40	43.20	87.40	52.20	91.30
100	84.00	99.80	87.85	99.90
	$\varphi_1 = 0$ and $\varphi_2 = -0.002$			
40	20.15	50.60	25.50	59.15
100	46.60	90.05	56.60	93.15
	$\varphi_1 = 0$ and $\varphi_2 = -0.001$			
40	8.45	14.65	10.75	19.35
100	13.50	30.80	18.45	38.30
	$\varphi_1 = 0$ and $\varphi_2 = 0$			
40	5.90	4.55	5.85	4.65
100	5.50	4.25	5.65	4.85
	$\varphi_1 = 0$ and $\varphi_2 = 0.001$			
40	8.75	13.75	10.00	18.30
100	13.35	32.60	17.50	38.50
	$\varphi_1 = 0$ and $\varphi_2 = 0.002$			
40	19.15	48.40	26.05	58.20
100	49.35	90.60	58.60	93.30
	$\varphi_1 = 0$ and $\varphi_2 = 0.003$			
40	43.30	86.90	50.95	91.15
100	85.60	99.90	90.25	100.00
	$\varphi_1 = 0$ and $\varphi_2 = 0.004$			
40	71.10	98.80	78.40	99.15
100	98.50	100.00	99.20	100.00
	$\varphi_1 = 0$ and $\varphi_2 = 0.005$			
40	87.35	99.95	91.40	100.00
100	99.90	100.00	99.95	100.00
	$\varphi_1 = 0$ and $\varphi_2 = 0.006$			
40	97.80	100.00	98.25	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = 0.007$			
40	97.80	100.00	98.25	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = 0.008$			
40	99.95	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = 0.009$			
40	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00
	$\varphi_1 = 0$ and $\varphi_2 = 0.01$			
40	100.00	100.00	100.00	100.00
100	100.00	100.00	100.00	100.00

Notes: *SupF* and *AveF* are *Sup* and *Ave*, F-tests of $\varphi_1 = \varphi_2 = 0$ in DGP4, with rejection frequencies computed at $\varphi_1 = 0$ and for $\varphi_2 = -0.01, 0.009, ..., 0.0, 0.001, ..., 0.009, 0.01$. Filtered pooled estimators are computed using $\mathbf{q}_{it} = (1, d_{it}, d_{i,t-1}, d_{i,t-2}, \Delta y_{i,t-1})'$ as the vector of filtering variables.

Table 9: MC findings for Bias(x100) and RMSE(x100) of the estimation of φ_1 and τ in DGP5

	Filtered pooled estimator			
	Bias (x100)		RMSE (x100)	
(N,T)	**40**	**100**	**40**	**100**
	φ_1 (true value $= -0.01$)			
40	0.0630	-0.0109	0.3197	0.0931
100	-0.0076	-0.0163	0.1074	0.0583
	τ (true value $= 0.80$)			
40	-2.69	-0.06	12.74	0.75
100	-0.02	-0.01	1.88	0.25

Notes: Filtered pooled estimators are computed using the vector of filtering variables, $\mathbf{q}_{it} = \left(1, d_{it}, d_{i,t-1}, d_{i,t-2}, \Delta y_{i,t-1}, \bar{\boldsymbol{\zeta}}_t', \bar{\boldsymbol{\zeta}}_{t-1}', ..., \bar{\boldsymbol{\zeta}}_{t-p}'\right)'$ where $\bar{\boldsymbol{\zeta}}_t$ is the arithmetic cross-sectional average of $\zeta_{it} = [d_{it}, \Delta y_{it}, g_{1it}(\tau)]'$ and p is the integer part of $0.5T^{1/3}$.

Table 10: Rejection frequencies of tests of $\varphi_1 = 0$ in DGP5, computed based on the filtered pooled estimator of φ_1

	Rejection rates (x100):									
	SupT		*AveT*		$\mathcal{T}$ (0.2)		$\mathcal{T}$ (0.5)		$\mathcal{T}$(0.9)	
(N,T)	46	100	46	100	46	100	46	100	46	100
	$\varphi_1 = -0.01$									
40	99.95	100.00	99.90	100.00	52.35	92.30	30.70	72.50	99.80	100.00
100	100.00	100.00	100.00	100.00	83.40	99.90	49.65	96.80	100.00	100.00
	$\varphi_1 = -0.009$									
40	99.95	100.00	99.90	100.00	48.10	88.00	25.80	63.25	99.25	100.00
100	100.00	100.00	100.00	100.00	78.05	99.75	45.00	93.55	100.00	100.00
	$\varphi_1 = -0.008$									
40	99.35	100.00	99.05	100.00	38.10	79.40	24.20	54.80	97.70	100.00
100	100.00	100.00	100.00	100.00	69.05	99.25	39.00	88.30	100.00	100.00
	$\varphi_1 = -0.007$									
40	98.30	100.00	96.75	100.00	32.95	69.85	21.70	46.40	93.25	99.95
100	100.00	100.00	100.00	100.00	61.10	96.85	34.50	81.95	99.90	100.00
	$\varphi_1 = -0.006$									
40	93.80	100.00	91.10	100.00	26.45	57.65	18.20	39.50	87.55	99.85
100	100.00	100.00	99.95	100.00	49.65	91.05	27.85	71.20	99.85	100.00
	$\varphi_1 = -0.005$									
40	84.45	99.95	82.95	99.35	23.40	46.35	16.75	31.15	75.50	98.95
100	99.75	100.00	99.10	100.00	40.90	82.05	21.80	57.80	97.95	100.00
	$\varphi_1 = -0.004$									
40	68.45	97.35	67.20	96.45	19.30	34.40	13.75	24.10	59.65	94.45
100	97.15	100.00	95.15	100.00	29.15	64.05	19.45	42.45	91.35	100.00
	$\varphi_1 = -0.003$									
40	48.75	82.85	49.10	81.75	16.40	24.40	13.35	17.45	40.90	77.05
100	82.50	99.85	80.05	99.35	22.55	45.80	15.80	28.75	73.80	98.80
	$\varphi_1 = -0.002$									
40	33.50	51.35	32.60	51.65	12.50	15.35	12.60	11.30	26.65	47.90
100	50.75	88.30	50.75	87.40	14.15	23.50	14.05	17.00	44.25	83.95
	$\varphi_1 = -0.001$									
40	19.55	22.10	19.05	22.30	11.70	10.85	10.40	10.25	13.65	18.60
100	26.05	36.65	27.10	39.25	13.10	12.45	11.75	9.60	21.45	35.25
	$\varphi_1 = 0$									
40	17.10	9.05	16.00	9.35	11.95	7.80	11.30	7.60	10.50	7.30
100	16.60	10.90	15.05	9.35	10.60	8.00	11.45	7.15	10.20	7.00
	$\varphi_1 = 0.001$									
40	21.15	20.50	20.90	22.20	11.55	9.90	12.35	9.20	14.25	20.00
100	24.85	38.55	26.40	39.80	11.05	12.15	12.90	11.65	19.55	35.55
	$\varphi_1 = 0.002$									
40	31.55	52.95	32.50	54.50	12.35	14.65	11.85	12.75	28.05	49.45
100	52.00	88.90	52.70	87.05	16.70	23.85	12.85	18.70	44.55	83.10
	$\varphi_1 = 0.003$									
40	49.75	86.10	50.80	83.60	14.70	22.65	11.60	18.25	43.80	80.35
100	82.15	99.90	80.50	99.45	20.25	41.45	15.55	30.90	73.45	98.90
	$\varphi_1 = 0.004$									
40	69.55	97.75	68.70	95.75	18.25	30.85	14.50	21.90	60.35	94.50
100	97.10	100.00	96.30	100.00	28.55	60.70	20.50	49.30	92.20	100.00
	$\varphi_1 = 0.005$									
40	84.35	99.90	83.40	99.75	22.50	43.45	18.15	33.35	76.35	99.10
100	99.75	100.00	99.50	100.00	39.30	77.50	25.05	62.45	98.05	100.00
	$\varphi_1 = 0.006$									
40	93.45	100.00	91.55	100.00	28.30	56.20	19.55	43.00	86.95	99.95
100	100.00	100.00	99.95	100.00	48.50	88.20	31.00	77.95	99.60	100.00
	$\varphi_1 = 0.007$									
40	97.85	100.00	96.70	100.00	29.45	65.25	21.90	54.20	93.55	100.00
100	100.00	100.00	100.00	100.00	57.60	95.20	34.80	85.95	99.90	100.00
	$\varphi_1 = 0.008$									
40	99.55	100.00	99.10	100.00	36.85	76.05	25.50	63.00	97.20	100.00
100	100.00	100.00	100.00	100.00	64.85	97.55	45.05	92.30	100.00	100.00
	$\varphi_1 = 0.009$									
40	99.90	100.00	99.85	100.00	43.95	83.70	29.60	71.55	98.60	100.00
100	100.00	100.00	100.00	100.00	73.70	99.35	48.20	97.25	100.00	100.00
	$\varphi_1 = 0.01$									
40	99.95	100.00	99.80	100.00	50.20	86.55	33.55	78.30	99.70	100.00
100	100.00	100.00	100.00	100.00	80.55	99.70	56.65	98.10	100.00	100.00

Notes: *SupT* and *AveT* are *Sup* and *Ave*, t-tests of $\varphi_1 = 0$ in DGP5, with rejection frequencies computed at $\varphi_1 = -0.01, 0.009, ..., 0.0, 0.001, ..., 0.009, 0.01$. $\mathcal{T}(\tau)$ is the t-test of the threshold effect ($\varphi_1 = 0$) computed for three *a priori* selected values of τ, $\tau = 0.2, 0.5$ and 0.9. Filtered pooled estimators are computed using the vector of filtering variables, $\mathbf{q}_{it} = \left(1, d_{it}, d_{i,t-1}, d_{i,t-2}, \Delta y_{i,t-1}, \bar{\boldsymbol{\zeta}}'_t, \bar{\boldsymbol{\zeta}}'_{t-1}, ..., \bar{\boldsymbol{\zeta}}'_{t-p}\right)'$ where $\bar{\boldsymbol{\zeta}}_t$ is the arithmetic cross-sectional average of $\zeta_{it} = [d_{it}, \Delta y_{it}, g_{1it}(\tau)]'$ and p is the integer part of $0.5T^{1/3}$.

Table 11: MC findings for the estimation of φ_1 and τ in DGP6

Rejection rates for $Sup\mathcal{T}$ and $Ave\mathcal{T}$ tests, and bias(x100) and RMSE(x100) for estimates of φ_1

	Without CS augmentation				**With CS augmentation**			
(N,T)	**40**	**100**	**40**	**100**	**40**	**100**	**40**	**100**
	Rejection rates of $Sup\mathcal{T}$ and $Sup\mathcal{T}$ tests							
	$Sup\mathcal{T}$		$Ave\mathcal{T}$		$Sup\mathcal{T}$		$Ave\mathcal{T}$	
40	63.45	79.60	65.55	81.35	12.10	7.55	9.90	7.65
100	83.25	92.05	83.45	92.90	9.95	7.65	9.20	6.60
	Bias(x100) and RMSE(x100) for estimates of φ_1							
	Bias (x100)		RMSE (x100)		Bias (x100)		RMSE (x100)	
40	0.2344	0.2211	1.1033	0.8628	0.0000	-0.0041	0.2258	0.1428
100	0.3034	0.3725	1.0245	0.8554	0.0008	0.0004	0.1375	0.0863

Notes: Filtered pooled estimators without cross-section (CS) augmentation are computed using $\mathbf{q}_{it} = (1, d_{it})'$ as the vector of filtering variables, and the filtered pooled estimators with CS augmentation are computed using the vector of filtering variables, $\mathbf{q}_{it} = \left(1, d_{it}, , \bar{\boldsymbol{\zeta}}_t', \bar{\boldsymbol{\zeta}}_{t-1}'\right)'$ where $\bar{\boldsymbol{\zeta}}_t$ is the arithmetic cross-sectional average of $\zeta_{it} = [d_{it}, \Delta y_{it}, g_{1it}(\tau)]'$.

Part B: Additional Empirical Results

Table 1: Tests of debt-threshold effects for *advanced economies* (robustness to the inclusion of inflation in the regressions), 1966-2010

	ARDL			DL				CS-ARDL		CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau)=I\left[d_{it}>\ln(\tau)\right]$ **and** $g_2(d_{it},\tau)=I\left[d_{it}>\ln(\tau)\right]\times\max\left(0,\Delta d_{it}\right)$													
$\widehat{\tau}$	0.80	0.60	0.60	0.80	0.80	0.80	0.80	0.10	0.10	0.10	0.10	0.10	0.20
$SupF$	20.21‡	19.82‡	20.5‡	18.74‡	24.44‡	20.2‡	22.09‡	4.52	12.19	10.03	4.54	5.02	12.12
$AveF$	11.7‡	11.57‡	10.53‡	11.47‡	13.11‡	10.38‡	9.85‡	2.14	3.0	4.39†	2.13	2.69	6.16*
CD	18.16	14.41	14.13	24.01	18.91	15.70	14.02	4.40	2.70	10.77	6.86	3.44	3.10
(b) Regressions with threshold variable $g_1(d_{it},\tau)=I\left[d_{it}>\ln(\tau)\right]$													
$\widehat{\tau}$	0.80	0.80	0.80	0.80	0.80	0.80	1.10	0.20	0.90	0.20	0.90	0.90	1.00
$Sup\mathcal{T}$	3.18†	2.73	2.43	3.98‡	4.24‡	3.98‡	3.74†	2.08	3.1	2.26	1.94	2.34	2.56
$Ave\mathcal{T}$	1.8‡	1.3†	0.84	2.95‡	3.07‡	2.63‡	2.19‡	1.21	1.14	1.19*	1.02	1.07	1.45
CD	18.76	14.90	14.47	24.67	20.10	16.37	14.89	7.07	4.30	10.98	6.48	4.74	4.88
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau)=I\left[d_{it}>\ln(\tau)\right]\times\max\left(0,\Delta d_{it}\right)$													
$\widehat{\tau}$	0.60	0.60	0.60	0.80	0.80	0.80	0.80	0.10	0.10	1.1	0.80	0.10	0.20
$Sup\mathcal{T}$	4.49‡	4.42‡	4.16‡	3.97‡	4.24‡	4.13‡	4.53‡	2.07	2.44	3.39†	2.55	2.4	3.62*
$Ave\mathcal{T}$	3.11‡	3.13‡	2.98‡	2.48‡	2.66‡	2.53‡	2.75‡	1.21	1.23	1.85‡	1.49†	1.57†	2.15‡
CD	18.77	14.45	14.22	24.56	19.58	16.21	14.03	3.77	2.85	7.36	6.91	3.21	2.23

Notes: In addition to Δd_{it}, inflation (π_{it}) and its lagged values are included as regressors in the ARDL and DL specifications, (22)–(23), while the CS-ARDL and CS-DL specifications, (24)–(25), also include the cross-sectional averages of π_{it} and its lagged values. Panel (a) reports the $SupF$ and $AveF$ test statistics for the joint statistical significance of both threshold variables $\left[g_1\left(d_{it},\tau\right)\text{ and }g_2\left(d_{it},\tau\right)\right]$, while panel (b) and (c) reports the $Sup\mathcal{T}$ and $Ave\mathcal{T}$ test statistics for the statistical significance of the simple threshold variable $g_1\left(d_{it},\tau\right)$, and the interactive threshold variable, $g_2\left(d_{it},\tau\right)$, respectively. Statistical significance of the Sup and Ave test statistics is denoted by *, † and ‡, at 10%, 5% and 1% level, respectively. CD is the cross-section dependence test statistic of (Pesaran 2004).

Table 2: Tests of debt-threshold effects for *developing economies* (robustness to the inclusion of inflation in the regressions), 1966-2010

	ARDL			DL				CS-ARDL		CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max(0, \Delta d_{it})$													
$\widehat{\tau}$	0.50	0.50	0.20	0.50	0.50	0.30	0.30	0.20	0.20	0.20	0.40	0.30	0.30
SupF	16.33†	14.94†	14.87*	12.15†	8.77	10.96	12.68	9.42	5.98	10.06	8.21	6.92	7.68
AveF	6.61‡	5.38‡	4.71*	6.43‡	5.16‡	4.35†	6.73‡	3.45	2.92	3.42	3.34	3.31	3.16
CD	6.27	6.14	4.54	6.85	6.33	4.53	4.71	-1.53	-0.58	-1.77	-1.32	-0.94	-1.60
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$													
$\widehat{\tau}$	0.50	0.50	0.50	0.50	0.50	0.30	0.50	0.40	0.40	0.50	0.40	0.50	0.50
$Sup\mathcal{T}$	2.1	1.74	1.77	2.24	2.5	2.89	3.38†	2.37	2.39	2.35	2.34	2.73	2.81
$Ave\mathcal{T}$	1.18*	1.05	0.74	1.45‡	1.4†	1.24*	1.89‡	1.29	1.47*	1.29†	1.47†	1.54†	1.36
CD	6.54	6.49	5.05	7.08	6.57	5.35	5.52	-1.33	0.11	-1.80	-1.49	-1.18	-2.19
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max(0, \Delta d_{it})$													
$\widehat{\tau}$	0.20	0.50	0.20	0.50	0.50	0.40	0.50	0.60	0.50	0.60	0.60	0.60	0.80
$Sup\mathcal{T}$	3.92‡	3.83†	3.15	3.54‡	2.98*	3.1*	3.46†	2.53	2.39	2.01	2.15	2.26	1.32
$Ave\mathcal{T}$	2.38‡	2.1‡	1.77‡	2.41‡	2.08‡	1.72‡	2.07‡	0.76	0.59	0.8	0.7	0.61	0.45
CD	6.00	6.18	4.39	7.12	6.74	5.04	5.23	-1.48	-0.46	-1.63	-1.31	-1.34	-1.83

Notes: See the notes to Table 1.

Table 3: Mean group estimates of the long-run effects of public debt and inflation on output growth for *advanced economies*, 1966-2010

	ARDL			DL				CS-ARDL		CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$													
$\widehat{\phi}_{\Delta d}$	-0.052‡	-0.050‡	-0.051‡	-0.113‡	-0.075‡	-0.089‡	-0.060‡	-0.065‡	-0.091‡	-0.078‡	-0.075‡	-0.071‡	-0.015
	(0.020)	(0.024)	(0.019)	(0.015)	(0.014)	(0.017)	(0.020)	(0.023)	(0.032)	(0.021)	(0.021)	(0.027)	(0.023)
$\widehat{\phi}_{\pi}$	-0.048*	0.026	0.037	-0.012	-0.025	0.037	0.048	-0.155‡	-0.148‡	-0.136‡	-0.131‡	-0.109‡	-0.151†
	(0.027)	(0.031)	(0.033)	(0.037)	(0.039)	(0.044)	(0.039)	(0.039)	(0.050)	(0.028)	(0.047)	(0.052)	(0.068)
CD	18.16	14.41	14.13	24.01	18.91	15.70	14.02	4.40	2.70	10.77	6.86	3.44	3.10
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$													
$\widehat{\phi}_{\Delta d}$	-0.069‡	-0.081‡	-0.073‡	-0.119‡	-0.082‡	-0.093‡	-0.080‡	-0.111‡	-0.114‡	-0.102‡	-0.114‡	-0.115‡	-0.075‡
	(0.018)	(0.020)	(0.019)	(0.017)	(0.014)	(0.018)	(0.020)	(0.023)	(0.030)	(0.016)	(0.022)	(0.025)	(0.023)
$\widehat{\phi}_{\pi}$	-0.049	0.024	0.034	-0.015	-0.024	0.035	0.092†	-0.167‡	-0.250‡	-0.132‡	-0.215‡	-0.246‡	-0.371‡
	(0.034)	(0.041)	(0.042)	(0.038)	(0.039)	(0.045)	(0.043)	(0.041)	(0.052)	(0.030)	(0.038)	(0.057)	(0.077)
CD	18.76	14.90	14.47	24.67	20.10	16.37	14.89	7.07	4.30	10.98	6.48	4.74	4.88
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$													
$\widehat{\phi}_{\Delta d}$	-0.045†	-0.053†	-0.053‡	-0.104‡	-0.062‡	-0.078‡	-0.060‡	-0.064‡	-0.080†	-0.084‡	-0.086‡	-0.069‡	-0.024
	(0.020)	(0.023)	(0.019)	(0.016)	(0.017)	(0.020)	(0.021)	(0.020)	(0.031)	(0.015)	(0.018)	(0.025)	(0.021)
$\widehat{\phi}_{\pi}$	-0.058†	0.011	0.017	0.005	-0.006	0.054	0.055	-0.149‡	-0.120†	-0.188‡	-0.205‡	-0.087*	-0.141‡
	(0.027)	(0.030)	(0.032)	(0.034)	(0.035)	(0.042)	(0.039)	(0.040)	(0.047)	(0.030)	(0.040)	(0.052)	(0.050)
CD	18.77	14.45	14.22	24.56	19.58	16.21	14.03	3.77	2.85	7.36	6.91	3.21	2.23
(d) Regressions without threshold variables													
$\widehat{\phi}_{\Delta d}$	-0.069‡	-0.093‡	-0.090‡	-0.087‡	-0.101‡	-0.086‡	-0.085‡	-0.099‡	-0.105‡	-0.111‡	-0.113‡	-0.086‡	-0.084‡
	(0.021)	(0.022)	(0.018)	(0.017)	(0.020)	(0.019)	(0.017)	(0.023)	(0.029)	(0.021)	(0.025)	(0.020)	(0.021)
$\widehat{\phi}_{\pi}$	-0.010	0.078*	0.085†	0.020	0.086*	0.098†	0.107†	-0.153‡	-0.158‡	-0.104‡	-0.117†	-0.147‡	-0.129†
	(0.032)	(0.040)	(0.041)	(0.037)	(0.046)	(0.046)	(0.048)	(0.040)	(0.047)	(0.038)	(0.053)	(0.050)	(0.063)
CD	19.55	15.21	14.80	21.80	17.62	16.01	15.57	6.38	5.25	9.02	6.76	6.56	8.29

Notes: In addition to Δd_{it}, inflation (π_{it}) and its lagged values are included as regressors in the ARDL and DL specifications, (22)–(23), while the CS-ARDL and CS-DL specifications, (24)–(25), also include the cross-sectional averages of π_{it} and its lagged values. Statistical significance is denoted by *, † and ‡, at 10%, 5% and 1% level, respectively.

Table 4: Mean group estimates of the long-run effects of public debt and inflation on output growth for *developing economies*, 1966-2010

	ARDL			DL				CS-ARDL		CS-DL			
lags:	(1,1)	(2,2)	(3,3)	p=0	p=1	p=2	p=3	(1,1,1)	(2,2,2)	p=0	p=1	p=2	p=3
(a) Regressions with threshold variables: $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$ **and** $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$													
$\widehat{\phi}_{\Delta d}$	-0.060‡	-0.047‡	-0.033*	-0.069‡	-0.068‡	-0.049‡	-0.059‡	-0.050‡	-0.059‡	-0.054‡	-0.061‡	-0.070‡	-0.080†
	(0.014)	(0.015)	(0.019)	(0.013)	(0.013)	(0.012)	(0.016)	(0.013)	(0.015)	(0.014)	(0.012)	(0.017)	(0.033)
$\widehat{\phi}_{\pi}$	-0.060*	-0.024	0.012	-0.053*	-0.049*	-0.009	0.019	-0.049*	-0.017	-0.069†	-0.057†	-0.059	-0.019
	(0.033)	(0.038)	(0.070)	(0.030)	(0.027)	(0.038)	(0.038)	(0.026)	(0.042)	(0.029)	(0.026)	(0.043)	(0.053)
CD	6.27	6.14	4.54	6.85	6.33	4.53	4.71	-1.53	-0.58	-1.77	-1.32	-0.94	-1.60
(b) Regressions with threshold variable $g_1(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right]$													
$\widehat{\phi}_{\Delta d}$	-0.068‡	-0.058‡	-0.071‡	-0.075‡	-0.075‡	-0.077‡	-0.077‡	-0.068‡	-0.083‡	-0.074‡	-0.070‡	-0.082‡	-0.091‡
	(0.013)	(0.015)	(0.022)	(0.014)	(0.012)	(0.012)	(0.017)	(0.014)	(0.018)	(0.016)	(0.014)	(0.017)	(0.033)
$\widehat{\phi}_{\pi}$	-0.061*	-0.026	0.010	-0.053*	-0.049*	-0.018	-0.007	-0.053*	-0.026	-0.080‡	-0.059*	-0.067	-0.035
	(0.032)	(0.038)	(0.063)	(0.030)	(0.027)	(0.036)	(0.036)	(0.031)	(0.046)	(0.028)	(0.030)	(0.044)	(0.051)
CD	6.54	6.49	5.05	7.08	6.57	5.35	5.52	-1.33	0.11	-1.80	-1.49	-1.18	-2.19
(c) Regressions with interactive threshold variable $g_2(d_{it},\tau) = I\left[d_{it} > \ln(\tau)\right] \times \max\left(0, \Delta d_{it}\right)$													
$\widehat{\phi}_{\Delta d}$	-0.032†	-0.047‡	-0.031	-0.067‡	-0.064‡	-0.032‡	-0.051‡	-0.075‡	-0.062‡	-0.073‡	-0.071‡	-0.071‡	-0.065†
	(0.013)	(0.015)	(0.019)	(0.013)	(0.013)	(0.012)	(0.017)	(0.013)	(0.014)	(0.016)	(0.013)	(0.015)	(0.033)
$\widehat{\phi}_{\pi}$	-0.063†	-0.025	0.017	-0.053*	-0.048*	-0.018	0.002	-0.070†	-0.042	-0.073‡	-0.060†	-0.057	-0.029
	(0.032)	(0.038)	(0.068)	(0.030)	(0.027)	(0.039)	(0.039)	(0.027)	(0.041)	(0.028)	(0.026)	(0.040)	(0.047)
CD	6.00	6.18	4.39	7.12	6.74	5.04	5.23	-1.48	-0.46	-1.63	-1.31	-1.34	-1.83
(d) Regressions without threshold variables													
$\widehat{\phi}_{\Delta d}$	-0.070‡	-0.062‡	-0.077‡	-0.074‡	-0.065‡	-0.068‡	-0.057‡	-0.072‡	-0.076‡	-0.071‡	-0.071‡	-0.078†	-0.037
	(0.014)	(0.015)	(0.022)	(0.013)	(0.014)	(0.017)	(0.020)	(0.016)	(0.018)	(0.015)	(0.019)	(0.033)	(0.037)
$\widehat{\phi}_{\pi}$	-0.064*	-0.030	-0.000	-0.050*	-0.028	-0.005	-0.029	-0.072*	-0.041	-0.050	-0.046	-0.030	-0.120*
	(0.033)	(0.041)	(0.067)	(0.028)	(0.037)	(0.039)	(0.038)	(0.038)	(0.046)	(0.031)	(0.046)	(0.060)	(0.070)
CD	7.28	6.98	5.17	7.96	6.92	6.91	5.81	-1.05	0.27	-0.65	-0.24	-1.04	0.15

Notes: See notes to Table 3.

References

Chudik, A., K. Mohaddes, M. H. Pesaran, and M. Raissi (2015). Is There a Debt-threshold Effect on Output Growth? Federal Reserve Bank of Dallas, Globalization and Monetary Policy Institute Working Paper forthcoming.

Pesaran, M. H. (2004). General Diagnostic Tests for Cross Section Dependence in Panels. *IZA Discussion Paper No. 1240*.