Reconstructing Economics: Agent Based Models and Complexity

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1. Introductive remarks*

In 1803 Louis Poinsot, a French physicist, wrote a book of great success, *Elements de Statique*, which was destined to have practical and social influences unimaginable to the same author. All this is due to the work of Leon Walras who took as a reference the system of simultaneous and interdependent equations of Poinsot with the introduction of the auctioneer, a device that allows to reduce the economic agents to atoms, devoid of any phenomenon of learning or strategic behavior. We were, around 1870, in the midst of classical mechanics and reductionism. The physics of interacting microelements was still to come, and economics found it trapped in the "equilibrated" view of Walras. And from there, the union of axiomatism and non-falsifiability led to the Lakatosian degeneration of the paradigm of mainstream economic theory.

The increasingly evident crisis of the dominant paradigm is manifested also through the "support" offered by other disciplines, from biology to chemistry, from neurology to physics. The contribution made to economic research from econophysics consists especially in an approach that makes extensive use of the experimental methodology and that operates on the data, often of very high frequency, related to the real markets, deriving empirical regularities (not laws) [1] and phenomenological models. Statistical physics provides useful tools to analyze systems composed of many heterogeneous agents (atoms) that interact in obedience to microscopic law. Unlike economic agents, however, atoms are not able to adopt either learning or strategic behavior, both of which are derived from interaction.

Each schematization, as long as it makes clear the logical scheme of the author, involves a loss of information. What I propose below is with no exception, although it has, in my opinion, an immediate and intelligible interpretation. Economic theory can be divided, for our purposes, in:

- A branch which is not dealing with direct interactions between agents, with the assumption of
    - A representative agent (RA), or of
    - Heterogeneous agents without infra-groups dynamics;
- A branch analyzing the issue of heterogeneous interacting agents (HIA) using Agent Based Models (ABM), which is separated into two approaches:

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* Leonardo Bargigli and Gabriele Tedeschi are partly responsible of what you read.

[1] I believe that the epistemological status of the hard sciences differs radically from that of the soft sciences. Consider, for example, a round of betting on "how long does it take for a certain object pushed out of a plane to fall on the floor". Suppose now that the law of falling bodies is revealed to all punters. There are not bets anymore, but the physical law remains valid: in the soft sciences, the discovery of a "law" means that the regularity disappears through learning.

- Agent based Computational Economics (ACE),
- Analytically Solvable HIA (ASHIA) models based on statistical physics or Markov chains.[2]

The difference between the 2 branches can be traced back to assumptions about information. If information is complete, there is no room for direct non-market interaction because agents do not need to increase their information through interaction. Some consequences follow: there is no coordination problem and thus no functioning pathologies. However, interaction, once introduced, involves non-linearity and externalities and, more generally, you lose the proportionality between cause and effect: small shocks may lead to large effects.

In my opinion, we must take into account the different nature of atoms and agents: this involves the transcending of the methodology proposed in statistical physics and the transition to the economics of complexity (: "*Imagine how hard physics would be if atoms could think*", Murray Gell-Mann, reported by Page, 1999). The main consequence of learning, or of learning atoms as stated in the quotation of Gell-Mann, is that the tools of physics cannot be translated *sic et simpliciter* in economics. The contributions of Foley (1994) and Aoki (1996) have introduced in economics the possibility to treat analytically the issue of many heterogeneous and interacting agents and thus to micro-founded aggregate behavior without recurring to the heroic but scientifically ridiculous hypothesis of a *representative agent*.[3] With this achievement we depart from the economics of mechanical equilibrium of the Poinsot-Walras setting to the economics of equilibrium probability distributions, in which the single agent can find herself outside of equilibrium and the system becomes complex.

A crucial aspect of the complexity approach is how interacting elements produce aggregate patterns that those elements in turn react to.[4] This leads to the emergence of aggregate properties and structures that cannot be guessed by simply looking at individual behaviour. It has been argued (Saari, 1995) that complexity is ubiquitous in economic problems (although this is rarely acknowledged in economic modelling), since (i) the economy is inherently characterized by the direct interaction of individuals, and (ii) these individuals have cognitive abilities, e.g. they form expectations on aggregate outcomes and base their behaviour upon them.[5]

In a nutshell, the passage from *economics* to the *economic complexity* will coincide (I will argue in the following) with the passage

- from an axiomatic discipline (what actually economics is) toward a falsifiable science (falsifiable at different levels of aggregation);
- an economic policy in which *one size does not fit all*;.

[2] With the introduction of the learning mechanism, Landini et al. (2012b) present a model that goes beyond the tools of statistical mechanics, opening the path for its applicability to social science, to economics in particular. After having derived analytic functions, they model learning agents as a finite memory Markov chain, and subsequently derive the corresponding master equation that describes the evolution of the population of the same agents. On modeling ABM as Markov chain see Gintis, 2012.

[3] Note also that the possibility of interaction between agents can address the problem of coordination failures (that is, for example, crises and fluctuations).

[4] A review of applications of the complexity theory to economics in Rosser, 1999, 2003; Arthur, 2000; Beinhocker, 2006b; Epstein, 2006b; Miller and Page, 2006; Kirman, 2011.

[5] Agent Based complexity theory should not be confused with general systems theory, a holistic approach developed in the 1950s and 1960s that in its most radical form argued that everything affects everything else: according to systems theory "phenomena that appear to have simple causes, such as unemployment, actually have a variety of complex causes - complex in the sense that the causes are interrelated, nonlinear, and difficult to determine" (Phelan, 2001). Conversely, the complexity approach looks for *simple* rules that underpin complexity.

2. From classical mechanics economics to...

2.1 The current crisis has carried the majority of economists to reflect on the state of economic theory. Unreliable economic models have not provoked the crisis, but they have not been able to prevent or even to forecast it. In particular, those reductionism models populated by a perfectly rational and fully informed representative agent turned out to be extremely fallacious (Kirman, 2009; Colander et al., 2009; but see Lucas, 2003, and Blanchard, 2009). Although the mainstream approach is still predominant, its internal coherence and ability in explaining the empirical evidence are increasingly questioned.

The causes of the present state of economic theory date back to the mid of the XVIII century (Mirowski, 1989), when some of the Western economies were transformed by the technological progress which lead to the industrial revolution. This was one century after the Newtonian revolution in physics: from the small apple to the enormous planets, all objects seemed to obey the simple *natural* law of gravitation. It was therefore immediate for a new figure of social scientist, the economist, to borrow the method (*mathematics*) of the most successful hard science, physics, allowing for the mutation of *political economy* into *economics*. It was (and still is) the mechanical physics of the XVII century, which ruled economics. From then on, economics has lived its own evolution based on the assumptions in classical physics (reductionism, determinism and mechanicism).

The ideas of natural laws and equilibrium have been transplanted into economics *sic et simpliciter*. As a consequence of the adoption of the classical mechanics paradigm, the difference between micro and macro was analysed under a reductionism approach. In such a setting, aggregation is simply the process of summing up market outcomes of individual entities to obtain economy-wide totals. This means that there is no difference between micro and macro: the dynamics of the whole is nothing but a summation of the dynamics of its components. This approach does not take into consideration that there might be two-way interdependencies between the agents and the aggregate properties of the system: interacting elements produce aggregate patterns that those elements in turn react to. What macroeconomists typically fail to realize is that the correct procedure of aggregation is not a sum: this is when emergence (i.e. the arising of complex structures from simple individual rules: Hayek, 1948; Schelling, 1978) enters the drama. Physics taught us that considering the whole, as something more than its constitutive parts is not only a theoretical construction: it is how reality behaves. Empirical evidence, as well as experimental tests, shows that aggregation generates regularities, i.e. simple individual rules, when aggregated, produce statistical regularities or well-shaped aggregate functions: regularities emerge from individual "chaos" (Lavoie, 1989).[6]

The research program launched by the neoclassical school states that macroeconomics should be

[6] The concept of equilibrium is quite a dramatic example. In many economic models equilibrium is described as a state in which (individual and aggregate) demand equals supply. The notion of *statistical equilibrium*, in which the aggregate equilibrium is compatible with individual disequilibrium, is outside the box of tools of the mainstream economist. The same is true for the notion of *evolutionary equilibrium* (at an aggregate level) developed in biology, according to which an individual organism is in equilibrium *only when it is dead*. The equilibrium of a system no longer requires that every single element be in equilibrium by itself, but rather that the statistical distributions describing aggregate phenomena be stable, i.e. in "[...] a state of macroscopic equilibrium maintained by a large number of transitions in opposite directions" (Feller, 1957: 356). A consequence of the idea that macroscopic phenomena can emerge is that reductionism is wrong.

explicitly grounded on microfoundations.[7] According to the mainstream approach, this implies that economic phenomena at a macroscopic level should be explained as a summation of the activities undertaken by individual decision makers. The reduction of the degree of freedom, which is characteristic of the aggregation problem in physics, is ruled out: a rational agent with complete information can choose to implement the individually optimal behaviour, without additional constraints.

There are three main pillars of this approach:

- The precepts of the rational choice-theoretic tradition;
- The equilibrium concept of the Walrasian analysis; and
- The reductionist approach of classical physics.

The first two assumptions, which constitute the necessary conditions for reducing macro to micro, are logically flawed (and empirically unfounded), while rejection of the third opens the road to complexity.

Mainstream economics, like theology, is an axiomatic discipline. According to the supporters of this view, such an abstraction is necessary since the real world is complicated: rather than compromising the epistemic worth of economics, such assumptions are essential for economic knowledge. However, this argument does not invalidate the criticism of unrealistic assumptions (Rappaport, 1996). While it requires internal coherence, so that theorems can be logically deduced from a set of assumptions, it abstracts from external coherence between theoretical statements and empirical evidence. Of course, this implies an important epistemological detachment from falsifiable sciences like physics.

2.2 In setting the methodological stage for the Dynamic Stochastic General Equilibrium (DSGE) macroeconomic theory, Lucas and Sargent declared: "*An economy following a multivariate stochastic process is now routinely described as being in equilibrium, by which is meant nothing more that at each point in time (a) markets clears and (b) agents act in their own self-interest. This development, which stemmed mainly from the work of Arrow [...] and Debreu [...], implies that simply to look at any economic time series and conclude that it is disequilibrium phenomenon is a meaningless observation. [...] The key elements of these models are that agents are rational, reacting to policy changes in a way which is in their best interests privately, and that the impulses which trigger business fluctuations are mainly unanticipated shocks.*" (Lucas and Sargent, 1979: 7.)

The self-regulating order (now it would be called SOC: Self Organisation Criticality) of Adam Smith (1776) is transformed into a competitive General Equilibrium (GE) in the form elaborated in the 1870s by Walras, that is a configuration of (fully flexible) prices and plans of action such that, at those prices, all agents can carry out their chosen plans and, consequently, markets clear. In a continuous effort of generalization and analytical sophistication, modern (neoclassical) economists interested in building microfoundations for macroeconomics soon recurred to the refinement proposed in the 1950s by Arrow and Debreu (1954), who showed that also individual intertemporal (on an infinite horizon) optimization yields a GE, as soon as the economy is equipped with perfect price foresights for each future state of nature and a complete set of Arrow-securities markets (Arrow, 1964), all open at time zero and closed simultaneously. Whenever these conditions hold true, the GE is an allocation that maximizes a properly defined social welfare function, or the equilibrium is Pareto-efficient (*First Welfare Theorem*).

[7] "The most interesting recent developments in macroeconomic theory seem to me describable as the reincorporation of aggregative problems [...] within the general framework of 'microeconomic' theory. If these developments succeed, the term 'macroeconomic' will be simply disappear from use and the modifier 'micro' will become superfluous. We will simply speak, as did Smith [sic!], Marshall and Walras, of *economic theory*" (Lucas, 1987: 107-8).

The literature has pointed out several logical inconsistencies of the mainstream approach. Davis (2006) identifies three "impossibility results" which determine the breakdown of the mainstream, i.e. neoclassical, economics:

- Arrow's 1951 theorem showing that neoclassical theory is unable to explain social choice (Arrow, 1963);
- The Cambridge capital debate pointing out that mainstream theory is contradictory with respect to the concept of aggregate capital (Cohen and Harcourt, 2003); and
- The Sonnenschein (1972), Mantel (1974), Debreu (1974) results showing that the standard comparative static reasoning is inapplicable in general equilibrium models, i.e. that GE is neither unique nor locally stable under general conditions.

Moreover, by construction, in a GE all transactions are undertaken at the same equilibrium price vector. Regardless of the mechanism (Walras' or Edgeworth's assumption) one adopts, the GE model considers the formation of prices precedes the process of exchange, instead of being the result of it, through a *tatonnement* process occurring in a meta-time. Real markets work the other way round and operate in real time, so that the GE model cannot be considered as scientific explanation of real economic phenomena (Arrow, 1959).

It has been widely recognized since Debreu (1959) that integrating money in the theory of value represented by the GE model is at best problematic. Given that in a GE model actual transactions take place only after a price vector coordinating all trading plans has been freely found, money can be consistently introduced into the picture only if the logical keystone of the absence of transaction costs is abandoned. By the same token, since credit makes sense only if agents can sign contracts in which one side promises future delivery of goods or services to the other side, in equilibrium markets for debt are meaningless, both information conditions and information processing requirements are not properly defined, and bankruptcy can be safely ignored.

The very absence of money and credit is a consequence of the fact that in GE there is complete information and rationality, i.e. there is no time. The only role assigned to time in a GE model is, in fact, that of dating commodities. Products, technologies, and preferences are exogenously given and fixed from the outset. The convenient implication of banning out-of-equilibrium transactions is simply that of getting rid of any disturbing influence of intermediary modifications of endowments - and therefore of individual excess demands - on the final equilibrium outcome.

2.3 Although the RA framework has a long history, it becomes standard to build the microfoundation procedure only after Lucas' critique paper (1976). Mainstream models are characterized by an explicitly stated optimization problem of the RA, while the derived individual demand or supply curves are used to obtain the aggregate demand or supply curves. Even when the models allow for heterogeneity, interaction is generally absent (the so-called *weak interaction hypothesis*: Rioss Rull, 1995). The use of RA models should allow to avoid the Lucas critique, to provide microfoundations to macroeconomics, and, *ça va sans dire*, to build Walrasian general equilibrium models.

Since models with many heterogeneous interacting agents are complicated, economists assume the existence of a RA: a simplification that makes it easier to solve for the competitive equilibrium allocation, since direct interaction is ruled out by definitions. Unfortunately, as Hildenbrand and Kirman (1988) noted: "*There are no assumptions on isolated individuals, which will give us the properties of aggregate behaviour. We are reduced to making assumptions at the aggregate level, which cannot be justified, by the usual individualistic assumptions. This problem is usually avoided in the macroeconomic* *literature by assuming that the economy behaves like an individual. Such an assumption cannot be justified in the context of the standard model."*

The *equilibria* of general equilibrium models with a RA are characterized by a complete absence of trade and exchange, which is a counterfactual idea. Kirman (1992), Gallegati (1993) and Caballero (1992) show that RA models ignore valid aggregation concerns, by ignoring interaction and emergence, committing fallacy of composition (what in philosophy is called "fallacy of division", i.e. to attribute properties to a different level from which the property is observed: game theory offers a good case in point with the concept of Nash equilibrium, by assuming that social regularities come from the agent level equilibrium). Those authors provide examples in which the RA does not represent the individuals in the economy so that the reduction of a group of heterogeneous agents to RA is not just an analytical convenience, but "both unjustified and leads to conclusions which are usually misleading and often wrong" (Kirman, 1992).

A further result, which is a proof of the logical fallacy in bridging the micro to the macro is the *impossibility theorem* of Arrow: it shows that an ensemble of people, which has to collectively take a decision, cannot show the same rationality of an individual (Mas-Colell et al., 1995). Moreover, the standard econometric tools are based upon the assumption of a RA. If the economic system is populated by heterogeneous (non necessarily interacting) agents, then the problem of the microfoundation of macro econometrics becomes a central topic, since some issues (e.g., *co-integration*, *Granger-causality*, *impulse-response function of structural VAR*) lose their significance (Forni and Lippi, 1997).

All in all, we might say that the failure of the RA framework points out the *vacuum* of the mainstream microfoundation literature, which ignores interactions: no toolbox is available to connect the micro and the macro levels, beside the RA whose existence is at odds with the empirical evidence (Stoker, 1995; Blundell and Stoker, 2005) and the equilibrium theory as well (Kirman, 1992).

3. … the ABM approaches.

3.1 What characterizes a complex system is the notion of emergence, that is the spontaneous formation of self-organized structures at different layers of a hierarchical system configuration (Crutchfield, 1994). Rather, mainstream economics conceptualizes economic system as consisting of several identical and isolated components, each one being a copy of a RA. The aggregate solution can thus be obtained by means of a simple summation of the choices made by each optimizing agent. The RA device, of course, is a way of avoiding the problem of aggregation by eliminating heterogeneity. But heterogeneity is still there. If the macroeconomist takes it seriously, he/she has to derive aggregate quantities and their relationships from the analysis of the micro-behaviour of different agents. This is exactly the key point of the *complexity approach*: starting from the *micro-equation*s describing/representing the (optimal) choices of the economic units, what can we say about the *macro-equations*? Do they have the same functional form of the micro-equations (the *analogy principle*)? If not, how to derive the macro-theory?

However, it is hard to recognize the imprinting of methodological individualism in the RA paradigm, which claims that the whole society can be analyzed in terms of the behaviour of a single representative individual and forgets to apply to it the Lucas critique. On the other hand, focusing on aggregate phenomena arising "from the bottom up" (Epstein and Axtel, 1996) via the interaction of many different agents, ABM also adopts a holistic approach when it claims that these phenomena cannot be studied without looking at the entire context in which they are embedded. Indeed, holism is the idea that all the properties of a given system cannot be determined or explained by the sum of its component parts alone. Instead, the system as a whole determines in an important way that the parts behave.[8]

Also, ABM can be regarded as a bridge between methodological individualism and methodological holism. In ABM models aggregate outcomes (the "whole", e.g. the unemployment rate) are computed as the sum of individual characteristics (its "parts", e.g. individual employment status). However, aggregate behaviour can often be recognized as distinct from the behaviour of the comprising agents, leading to the discovery of emergent properties (Dosi et al., 2010; Gallegati et al., 2010). In this sense, the whole is more than - and different from - the sum of its parts. It might even be the case that the whole appears to act as if it followed a distinct logic, with its own goals and means, as in the example of a cartel of firms that act in order to influence the goods market price. From the outside, the "whole" appears no different from a new agent type (e.g. a family, a firm). The computational experiment that gives birth to a new entity has been successful in "growing artificial societies from the bottom up" (Epstein and Axtell, 1996).

This *bottom-up* approach to complexity consists in "deducing the macroscopic objects (*macros*) and their phenomenological complex *ad-hoc* laws in terms of a multitude of elementary microscopic objects (*micros*) interacting by simple fundamental laws" (Solomon, 2007), and ABM provides a technique that allows to systematically follow the birth of these complex macroscopic phenomenology. The *macros* at a specific scale can become the *micros* at the next scale.

The ABM methodology is from bottom-up and is focused on the interaction of many heterogeneous interacting agents, which might produce a statistical equilibrium (Miller and Page 2006; Epstein 2006b; see also Batten, 2000, Wooldridge, 2002, Gilbert and Troitzsch, 2005 and Flake, 1998).

ABM is a methodology that allows to construct models with heterogeneous agents, based on simple behavioural rules and on interaction, where the resulting aggregate dynamics and empirical regularities are not known *a priori* and are not deducible from individual behaviour (G.Nicolis, C.Nicolis, 2007). ABM is characterized by two main tenets: (i) there is a multitude of objects that interact with each other and with the environment; (ii) the objects are autonomous, i.e. there is no central, or "top down" control over their behaviour.

The bottom-up approach models individual behaviour according to simple behavioural rules; agents are allowed to have *local interaction* and to change the *individual rule* (*through adaptation*) as well as the *interaction nodes*. By aggregating, some *statistical regularity* emerges, which cannot be inferred from individual behaviour *(self emerging regularities)*: this *emergent behaviour* feeds back to the individual level *(downward causation)* thus establishing a macro foundation of micro behaviour. As a consequence, each and every proposition may be falsified at *micro*, *meso* and *macro* levels. This approach opposes the axiomatic theory of economics, where the optimization procedure is the standard for a scientific, i.e. not ad-hoc, modelling procedure.[9]

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[8] The general principle of holism was concisely summarized by Aristotle in his *Metaphysics*: "The whole is more than the sum of its parts", a *manifesto* of the complexity approach.

[9] The agent-based methodology can also be viewed as a way to reconcile the two opposing philosophical perspectives of methodological individualism and holism. Having agents as unit of analysis, ABM is deeply rooted in methodological individualism, a philosophical method aimed at explaining and understanding broad society-wide developments as the aggregation of decisions by individuals (von Mises, 1949; Arrow, 1994). Methodological individualism suggests - in its most extreme (and erroneous) version – that a system can be understood by analyzing *separately* its constituents, the

3.2 Agent-based computational economics (ACE) is the area of computational economics that studies economic processes, including whole economies, as dynamic systems of interacting agents. As such, it falls in the paradigm of complex adaptive systems. In corresponding agent-based models, the "agents" are "computational objects modelled as interacting according to rules" over space and time, not real people. The rules are formulated to model behaviour and social interactions based on incentives and information.

The theoretical assumption of mathematical optimization by agents in equilibrium is replaced by the less restrictive postulate of agents with bounded rationality adapting to market forces. ACE models apply numerical methods of analysis to computer-based simulations of complex dynamic problems for which more conventional methods, such as theorem formulation, may not find ready use. Starting from initial conditions specified by the modeller, the computational economy evolves over time as its constituent agents repeatedly interact with each other, including learning from interactions. In these respects, ACE has been characterized as a bottom-up culture-dish approach to the study of economic systems.

The outcome of interaction is numerically computed. Since the interacting objects are autonomous, they are called "agents": "Agent-based Computational Economics is the computational study of economic processes modelled as dynamic systems of interacting agent" (Tesfatsion 2002, 2006; Gintis, 2007; Chen, 2012.)

Here "agent" refers broadly to a bundle of data and behavioral methods representing a constitutive part of a computationally constructed world. The availability of high-speed processors and the possibility to handle large amounts of data has undoubtedly contributed to the success of ACE models. One of the problems detected is related to parameter setting: with many degrees of freedom, as is often objected, every result becomes possible. The process of aggregation, in physics, takes away these degrees of freedom. The procedure of microfoundation in economics is very different from that used in physics. The latter starts from the micro-dynamics of the single particle, as expressed by the Liouville equation and, through the Master equation, ends up with the macroscopic equations. In the aggregation process, the dynamics of the agents lose their degrees of freedom and behave coherently in the aggregate. In mainstream economics, while the procedure is formally the same (from micro to macro), it is assumed that the dynamics of the agents are those of the aggregate. The reduction of the degree of freedom, which is characteristic of the aggregation problem in physics, is therefore ruled out: a rational agent with complete information can choose to implement the individually optimal behaviour, without additional constraints.

3.3 Following the example of statistical physics, Foley (1994), Aoki (1996, 2002), Aoki and Yoshikawa (2006) have proposed to aggregate statistically HIA.[10] In this way, one can obtain an analytical solution for populations of millions of agents that interact. Figure 1 represents the behavior of the same population - 1 million firms *vs.* 1 ME in the model of Landini et al. (2012a) - described respectively by the methods of ACE and of ASHIA. The correlation between the two is 0.87: note that the approximation results in an acceptable discrepancy, especially in front of the reduction in the number of equations: from 1 million to 1!

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reductionist approach that the "whole" is nothing but the "sum of its parts" (Descartes, 1637; Nagel, 1961). However, the ability to reduce everything to simple fundamental objects and laws does not imply the ability to start from those objects and laws and reconstruct the universe. In other terms, reductionism does not imply constructionism (Anderson, 1972).

[10] See also Di Guilmi et al. (2011); Landini et al. (2012a,b); Gallegati et al. (2006); Lux (2009).

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Until now, the analyses of systems with HIA have been limited to simulations. The ASHIA approach follows a quite different methodology, adopting analytical tools originally developed in statistical mechanics, and subsequently adopted by social disciplines. As the economy is populated by a very large number of heterogeneous and interacting agents, we cannot know which agent is in which condition at a given time and whether an agent will change its condition, but we can know the probability of a given state of the world. The basic idea consists of introducing a level of aggregation, obtained by grouping the agents in clusters according to a measurable variable. This state variable is chosen in such a way that the dynamics of the number of individuals in each cluster also define the evolution of the whole economy.

This study is made possible by specifying some general assumptions on the stochastic evolution of the occupation numbers, which can be modeled by means of the master equation (ME). The latter is a simple first-order differential equation, which quantifies the evolution through time of the probability of observing a given number of agents in a certain state. The process of reducing the vector of observations of a variable over a population to a single value (i. e. the computation of an average level of output for each dimensional bin) is defined as mean field approximation. The definitions of the mean-field variables and of the probabilities involve some level of interaction among agents.[11]

The ME allows us to derive an analytic device, which might have a strong impact on macroeconomic modeling. It proposes a solution to the problem of performing the aggregation

[11] Mean-field theory has been introduced in economics in different models by Brock and Durlauf, who show how mean-field interaction is able to generate a multiplicity of Nash-type equilibria.

when heterogeneity and nonlinearities are present, an issue, which is debated in the literature at least since the introduction of the exact aggregation. In fact, the ME is an analytical tool that allows us to interpret the aggregate dynamics by means of a (mechanical) statistical approach, that integrates the heterogeneity by considering a multiplicity of representative states of a system, and implements the interaction (as a mean field interaction) by means of the specification of the transition rates. In such a way, the problem of the aggregation of heterogeneous agents is originally solved, without resorting neither to the unrealistic simplifications and assumptions of mainstream theory, nor to the "black box" of computer simulations as in the standard ACE approach. Furthermore this methodology can effectively deal with the issue of agents' interaction, by functionally embodying it in the determination of the probabilistic transition rules (the transition rates), which change endogenously in time. As one can see, this development changes macroeconomic modeling: the empirical evidence can directly infer the representation of the micro-level, and the task of the researcher is just to identify a suitable set of relationships among the micro-variables[12].

Moreover, as demonstrated by the physics literature, the ME can be effectively employed to model analytically the dynamics of the topology of a network. Landini et al. (2012a), for instance, analyses the evolution of the degree distribution of a network. The solution of the ME yields the dynamics of the network degree, providing a synthetic and formal representation of the concentration in the market and, thus, of the fragility of the network.

Summarizing, the main contributions of ASHIA are:

- To provide a solution to the problem of aggregation in HIAs by using methods inspired by the statistical mechanics against the usual classical mechanics involved in RA based models;
- To propose a dynamic stochastic model for sub-populations of many HIAs interacting in an endogenous evolving network;
- To develop an analytic solution in which the model is defined as a deterministic ordinary differential equation describing the dynamics of network.

The dynamic stochastic aggregation is able to provide a complete and consistent analytical representation of the system, using numerical simulations only as a further verification of the results of the model.

Economic agents are not atoms, however: they have many more choices according to their preferences and endowments, but mostly according to rationality and information. There is only one-way to match *homo oeconomicus* with atom: perfect information and complete rationality. A person not with a sound mind can behave in a totally unpredictable way, while even a poorly informed can guess. In this process of "trials and errors" lies the process of learning, often modelled by complexity theorists, such as CAS.

Apart from the equivalence between atom and *homo oeconomicus*, we must consider some consequences, among which, in my opinion, the following three points are the most relevant:

- With learning, the contribution of ASHIA profoundly changes: it is no longer the mere transposition into economics of an instrument borrowed from statistical physics, since it is

[12] It is worth noticing that this approach overcomes another limit of the RA modeling, as the equilibrium is no more a fixed point in the space, but a probability distribution: a system can be in equilibrium even if its constitutive elements are not.

transformed in the direction of scientifically studying the society in which individual behaviour changes and it is changed by the aggregate context;

- To appreciate the systemic aspect of an economy, one has to analyze the individual as well the global characteristics of the system by analysing the individual nodes as agents and their links in the network;
- The social aspect of the economic atom is expressed in networks: the links between agents are established (e.g. to form credit linkages and/or to increase the information set) and are changed according to fitness (see Barabási and Albert (1999), Bianconi and Barabási (2001), Tedeschi et al. (2011)).

According to the mainstream approach, there is no direct interaction among economic units (for a pioneer, though neglected, contribution see Foellmer, 1974; see also Kirman, 2000). In the most extreme case, any individual strategy is excluded (*principle of excluded strategy*, according to Schumpeter, 1960) and agents are homogeneous. Small departures from the perfect information hypothesis are incoherent with the Arrow-Debreu general equilibrium model, as shown by Grossman and Stiglitz (1980), since they open the chance of having direct links among agents (Stiglitz, 1992). In particular, if prices convey information about the quality, there cannot be an equilibrium price as determined by the demand-supply schedule, since demand curves depend on the probability distribution of the supply (Grossman and Stiglitz, 1976: 98).

4. The role of economic policy

Economics was *political economy* before. According to classical economists, the economic "science" has to be used to control the real economies and steer them towards desirable outcomes. If one considers the economic system as an analogue of the physical one, it is quite obvious to look for *natural* economic policy prescriptions (*one policy fits all*). This is the approach of mainstream (neoclassical) economists.

There is a widespread opinion, well summarized by Brock and Colander (2000), that complexity does not add anything new to the toolbox of mainstream economic policy analysis. This view needs substantial corrections (see also the reflections by Durlauf, 1997). The complexity approach showed us that the age of certainty ended with the *non-equilibrium* revolution, exemplified by the works of Prigogine. Considering the economy as an evolving (*adaptive*) system we have to admit that our understanding is limited: there is no room for the Laplace' demon in complexity. Individual behavioural rules evolve according to their past performance: this provides a mechanism for an endogenous change of the environment. As a consequence the "rational expectation hypothesis" looses significance. However, agents are still rational in that they do what they can in order not to commit systematic errors (Lewontin and Levins, 2008). In this setting there is still room for policy intervention outside the mainstream myth of a neutral and optimal policy. Since emergent facts are transient phenomena, policy recommendations are less certain, and they should be institution dependent and historically oriented (Finch and Orillard, 2005). In particular, it has been emphasized that complex systems can either be extremely fragile and turbulent (a slight modification in some minor detail brings macroscopic changes), or relatively robust and stable: in such a context, policy prescriptions ought to be case sensitive.

Real economies are composed of millions of interacting agents, whose distribution is far from being stochastic or normal. As an example, figure 2 reports the distribution of firms' trade-credit relations in the electronic-equipment sector in Japan in 2003 (see De Masi et al., 2010). It is quite evident that there exists several hubs, i.e. firms with many connections: the distribution of the degree of connectivity is *scale free*, i.e. there are a lot of firms with 1 or 2 links, and quite a few firms with a lot of connections. Let us assume the Central Authority has to prevent a *financial collapse* of the system, or the spreading of a financial crisis (the so-called *domino effect*, see e.g. Krugman, 1998 and Stiglitz, 2002). Rather than looking at the "average" risk of bankruptcy (in power law distributions the mean may even not exist, *i.e.* there is an empirical mean, but it is not stable), using the latter as a measure of the stability of the system by means of a network analysis, the economy can be analyzed in terms of different interacting sub-systems and local intervention can be recommended to prevent failures and their spread.[13] Instead of a helicopter drop of liquidity, one can make "targeted" interventions to a given agent or sector of activity.

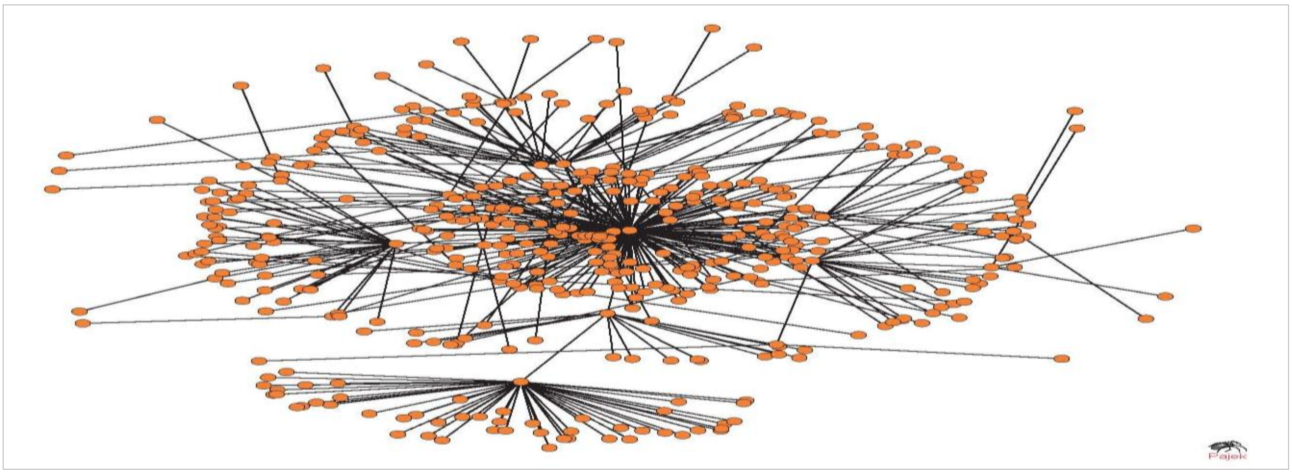

Figure 2. Distribution of firms' trade-credit relations in the electronic-equipment sector in Japan, 2003 (De Masi et al., 2010).

In this perspective, notions elaborated from network theory become very relevant, like resilience, which depicts the behaviour of network's structures following the removal of some nodes. In particular, whenever a vertex is removed from a network, the average distance among nodes increases and, as this process goes further, some nodes will be disconnected ultimately. Nodes can be removed in many ways. They may be attacked randomly or according to some of their intrinsic properties (such as their degree). Depending on the rules used to remove nodes, the network shows a different level of resilience. For instance, Albert and Barabasi (2000) show that social networks, usually highly right-skewed, are remarkably resistant to random attacks but extremely vulnerable to attacks targeted at nodes with the highest degree (hubs). To prove this claim, the authors remove nodes in decreasing order of their connectivity, showing that, as a small number of hubs are removed, the average distance of the scale-free network increases rapidly.

Network topology is relevant for systemic risk too. Credit relationships, which have acted as a major channel of contagion during the crisis, can be naturally conceived as networks in which nodes represent agents and links represent credit claims and liabilities. In particular, it becomes important to identify densely connected subsets of nodes within such networks, i.e. modules or communities. In fact, community structure is tightly related to the issue of diversification because, in a nutshell, the latter may be attained only where the former is suppressed. Since instead communities are likely to be ubiquitous in real economic networks, community detection provides a general approach to the analysis of contagious defaults. In fact, contagion is dependent on the

[13] For instance, Fujiwara (2008) shows how to calculate the probability of going bankrupt by *solo*, i.e. because of idiosyncratic elements, or *domino* effect, i.e. because of the failure or other agents with which there exist credit or commercial links.

geometric properties of the network with adjacency matrix representing (possibly weighted) connections, which on its part is related to the community structure of the network (Bargigli and Gallegati, 2012). Thus a community detection algorithm provides a general recipe to detect those areas of the financial system, which are most likely to be affected when some nodes are initially hit by shocks, without the need to specify in advance such shocks.

In a heterogeneous interacting agents environment, there is also room for an extension of the Lucas critique. It is well known that, since the underlying parameters are not policy invariant, any policy advice derived from large-scale econometric models that lack microfoundations would be misleading. The Lucas Critique implies that in order to predict the effect of a policy experiment, the so-called *deep parameters* (*preferences, technology* and *resource constraints*) that govern individual behaviour have to be modelled. Only in this case it is possible to predict the behaviour of individuals, conditional on the change in policy, and aggregate them to calculate the macroeconomic outcome. But here is the trick: aggregation is a sum only if interaction is ignored. If non-price interactions (or other non-linearities) are important, then the interaction between agents may produce very different outcomes. Mainstream models focus on analytical solvable solutions: to get them, they have to simplify the assumptions, e.g. using the RA approach or a Gaussian representation of heterogeneity. At the end, the main objective of these models is to fit the theory, not the empirical: how to explain, e.g., the scale-free network of the real economy by using the non-interacting network of the mainstream model? At a minimum, one should recognize that the mainstream approach is a very primitive framework and, as a consequence, the economic policy recommendations derived from it are far from being adequate prescriptions for the real world.

One of the traditional fields of applications of economic policy is *redistribution*. It should be clear that a sound policy analysis requires a framework built without the RA straight jacket. A redistributive economic policy has to take into account that individuals are different: not only they behave differently, e.g. with respect to saving propensities, but also they have different fortunes: the so-called *St. Thomas* (13:12) effect (*"to anyone who has, more will be given and he will grow rich; from anyone who has not, even what he has will be taken away"*), which is the road to Paradise for Catholics, and to the power-law distribution of income and wealth for the econophysicists.

Gaffeo et al. (2007) shows that there is a robust link between firms' size distribution, their growth rate, and GDP growth (see also Gabaix, 2011). This link determines the distributions of the amplitude frequency, size of recessions and expansion, *etc.* Aggregate firms' size distribution can be well approximated by a *power law* (Axtell, 2001; Gaffeo et al., 2003), while sector distribution is still right skewed, but without scale-free characteristics (Axtell et al., 2006). Firms' growth rates are far from being *normal*: in the central part of the distribution they are tent shaped with very fat tails. Moreover, empirical evidence shows that an inverse function of firms' age and size exists, and is proportional to financial fragility. In order to reduce the volatility of fluctuations, policy makers should act on the firms' size distribution, allowing for a growth of their capitalization, their financial solidity and wealth redistribution (Delli Gatti et al., 2004, 2005). Since these emerging facts are policy sensitive, if the aggregate parameters change the shape of the curve will shift as well.

Different from Keynesian economic policy, which theorizes aggregate economic policy tools, and different from mainstream neoclassical economics, which prescribes individual incentives because of the Lucas critique but ignores interaction which is a major but still neglected part of that critique, the ABM approach proposes a bottom up analysis. What generally comes out is not a "one-size-fits-all" policy since it depends on the general as well as the idiosyncratic economic conditions; moreover, it generally has to be conducted at different levels (from micro to meso to macro). In short, ABM can offer new answers to old unresolved questions, although it is still in a far too premature stage to offer definitive tools.

5. Future Directions

Depending on the scope of the analysis, it is generally convenient to stop at some scale in the way down to reconstruct aggregate and top-level dynamics "from the bottom up". When applied to economics, only a few levels (e.g. a *micro*, a *meso* and a *macro* level) are in general sufficient to provide a thorough understanding of the system. Defining the elementary units of analysis amounts to fixing the limits for the reductionist approach, which is not a priori discarded but rather integrated in the analysis. These units are in fact characterized by an inner structure that does not depend on the environment in which they are embedded. They can thus be analyzed separately.

The need for the ABM approach at any given scale is often linked to the existence of some underlying autocatalytic process at a lower level. Autocatalytic processes are dynamic processes with positive feedbacks, where the growth of some quantity is to some extent self-perpetuating, as in the case when it is proportional to its initial value.[14] However, the traditional analysis is static, and does not address how equilibrium out of several might be selected. Looking at the problem from the perspective of dynamic stochastic process, selection is explained in terms of one set of small historical events magnified by increasing returns.

Moreover, the existence of an autocatalytic process implies that looking at the average, or most probable, behaviour of the constituent units is not representative of the dynamics of the system: "autocatalyticity insures that the behaviour of the entire system is dominated by the elements with the highest auto-catalytic growth rate rather than by the typical or average element" (Solomon, 2007). In presence of autocatalytic processes, even a small amount of individual heterogeneity invalidates any description of the behaviour of the system in terms of its "average" element: "the real world is controlled as much by the *tails* of distributions as by means or averages. We need to free ourselves from *average* thinking" (Anderson, 1997).

The fact that autocatalytic dynamics are scale invariant (i.e. after a transformation that multiplies all the variables by a common factor) is a key to understanding the emergence of scale invariant distributions of these variables (e.g. power laws), at an aggregate level. The relevance of scale free distributions in economics (e.g. of firm size, wealth, income, etc.) is now extensively recognized (Brock, 1999), and has been the subject of through investigation in the econophysics literature (Mantegna and Stanley, 2000).

In the 4th edition of his *Principles*, Marshall wrote, "The Mecca of the economist is biology". What he meant to say was that, because economics deals with learning agents, evolution and change are the *granum salis* of our economic world. A theory built upon the issue of allocations of given quantities is not well equipped for the analysis of change. This allocation can be optimal only if there are no externalities (increasing returns, non-price interactions etc.) and information is complete, as the *invisible hand* shows. In the history of science, there is a passage from a view emphasizing centralised "intelligent design" to a view emphasizing *self organised criticality* (Bak, 1997), according to which a system with many heterogeneous interacting agents reaches a

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[14] The importance of positive feedback has been recognized in the literature on increasing returns, in particular with respect to the possibility of multiple *equilibria* (Semmler, 2005), since the time of Marshall.

statistical aggregate equilibrium, characterised by the appearance of some (often scale free) stable distributions. These distributions are no longer "optimal" or "efficient" according to some welfare criterion: they are simply the natural outcome of individual interaction.

Because of the above-mentioned internal and external inconsistencies of the mainstream approach, a growing strand of economists is now following a different methodology based upon the analysis of systems with many heterogeneous interacting agents. Their interaction leads to empirical regularities, which emerge from the system as a whole and cannot be identified by looking at any single agent in isolation: these emerging properties are, according to us, the main distinguishing feature of a complex system. The focus on interaction allows the scientist to abandon the heroic and unrealistic RA framework, in favour of the *science of complexity*. Complexity approach is a very tough line of research whose empirical results are very promising (see e.g., chapters 2-3 in Gaffeo et al., 2008). Modelling an agent-based economy however remains itself a complex and complicated adventure.

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